

Atmospheric Neutrinos

50 GeV to 100 TeV

Outline

- Phenomenology of atmospheric neutrinos
 - Solutions of cascade equation
 - Relation to muons
 - Importance of kaons
 - Angular distributions
 - Energy dependence
 - Primary spectrum
 - Hadronic models, scaling violation
- Muon charge ratio
 - Implications for neutrino/anti-neutrino ratio
 - Separate ν_μ and $\bar{\nu}_\mu$ fluxes
- Summary
 - Uncertainties

Cascade equation

$$\frac{dN_i(E_i, x, \theta)}{dX} = \underbrace{-\frac{N_i(E_i, X, \theta)}{\lambda_i(E_i)} - \frac{N_i(E_i, X, \theta)}{d_i(E_i)}}_{\text{Loss terms}} + \underbrace{\sum_{j=i}^J \int_{E_i}^{\infty} \left\{ \frac{d\sigma_{ji}(E_j, E_i)}{\sigma_j^{inel}(E_j) \lambda_j(E_j)} + \frac{R_{ji}}{d_j} \right\} N_j(E_j, X, \theta) dE_j}_{\text{Source terms}}$$

Interaction Decay
Interaction Decay

Boundary conditions:

$$\left[\begin{array}{l} N_i(E_0, 0, \theta) = \phi_N(E_0) \delta_{iN} \quad \text{for inclusive flux} \\ N_i(E_0, 0, \theta) = \delta(E_0 - AE_N) \delta_{iA} \quad \text{Air shower} \end{array} \right.$$

Matrix Cascade Equation (MCEq)* starts with a nucleon of energy E_0 and integrates in steps of dX for each θ with a matrix of 65 particle types and 8 energy bins per decade.

*<https://github.com/afedynitch/MCEq>

Analytic Approximations (AA)

For a power law primary spectrum + scaling of production σ , solutions for low and high energy have the forms:

$$\frac{dN_\ell}{dE_\ell} = \int_0^{X_0/\cos\theta} P_{M\ell}(E_\ell, X) dX \Big|_{E_\ell \ll \epsilon_M} \rightarrow \frac{N_0(E_\ell)}{1 - Z_{NN}} A_{M\ell}$$

$$\frac{dN_\ell}{dE_\ell} = \int_0^{X_0/\cos\theta} P_{M\ell}(E_\ell, X) dX \Big|_{E_\ell \gg \epsilon_M} \rightarrow \frac{N_0(E_\ell)}{1 - Z_{NN}} \left(\frac{\epsilon_M}{\cos\theta E_\ell} \right) \frac{A_{M\ell}}{B_{M\ell}}$$

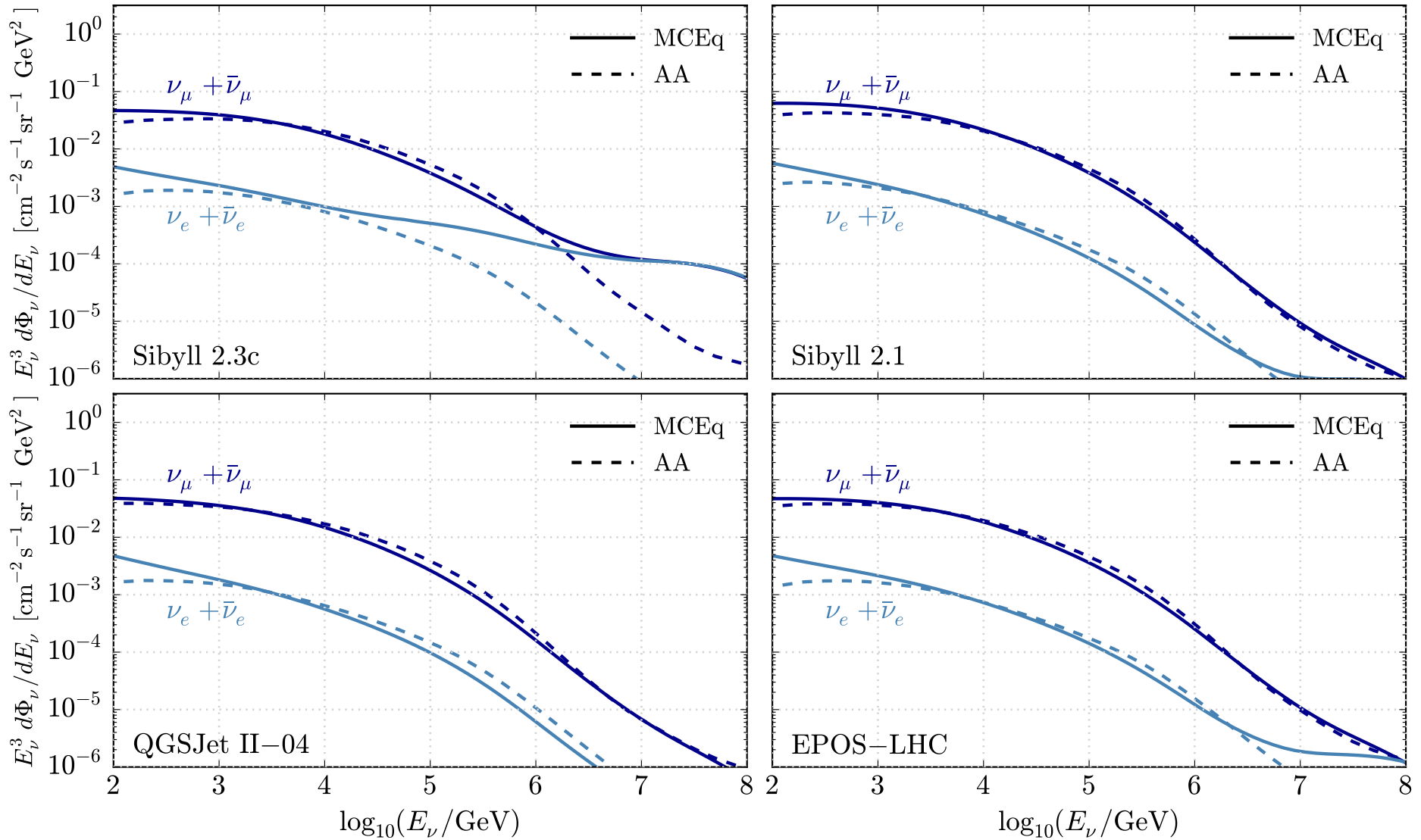
Combine low and high-energy forms in the approximation

$$\frac{dN_\ell}{dE_\ell} = \frac{N_0(E_\ell)}{1 - Z_{NN}} \left\{ \frac{A_{\pi\ell}}{1 + B_{\pi\ell} \cos\theta E_\ell/\epsilon_\pi} + \frac{A_{K\ell}}{1 + B_{K\ell} \cos\theta E_\ell/\epsilon_K} \right\}$$

$$A_{M\ell} = R_{M\ell} Z_{NM} Z_{M\ell}(\gamma) \quad B_{M\ell} = \frac{Z_{M\ell}(\gamma)}{Z_{M\ell}(\gamma + 1)} \frac{\Lambda_M - \Lambda_N}{\Lambda_M \ln(\Lambda_M/\Lambda_N)}$$

Compare methods

TG, D. Soldin, A. Crossman, A. Fedynitch
 ICRC 2019, arXiv:1910.08676



Two body decays of π^\pm & K^\pm

For a power-law primary spectrum of nucleons with integral spectral index $\gamma \approx 1.7$

$$Z_{\pi\mu} = \frac{(1 - r_\pi^{\gamma+2})}{(\gamma + 2)(1 - r_\pi)} \quad \text{and} \quad Z_{\pi\nu\mu} = \frac{(1 - r_\pi)^{\gamma+2}}{(\gamma + 2)(1 - r_\pi)}$$

$$r_\pi = \frac{m_\mu^2}{m_\pi^2} \approx 0.573 \quad r_K = \frac{m_\mu^2}{m_K^2} \approx 0.046$$

$$Z_{\pi\nu\mu} \ll Z_{K\nu\mu}$$

Critical energies and Z-factors

$$\epsilon_M = \frac{RT(X) Mc^2}{M_{\text{mol}} g c \tau_M} \quad \text{For } T = 220^\circ \text{ K} \left\{ \begin{array}{l} \epsilon_\pi = 115 \text{ GeV} \\ \epsilon_{K^\pm} = 857 \text{ GeV} \\ \epsilon_{D^\pm} \approx 3.7 \times 10^7 \text{ GeV} \end{array} \right.$$

$$Z_{NM}(E) = \int_E^\infty \frac{N_0(E') \sigma(E')}{N_0(E) \sigma(E)} \frac{dn_{NM}(E', E)}{dE} dE' \rightarrow \int_0^1 x^\gamma \frac{dn_{NM}(x)}{dx} dx$$



Energy-dependent Z-factors

Gondolo, Ingelman, Thunman, Astropart. Phys. 5 (1996) 309



Scaling

How the ν_μ spectrum steepens

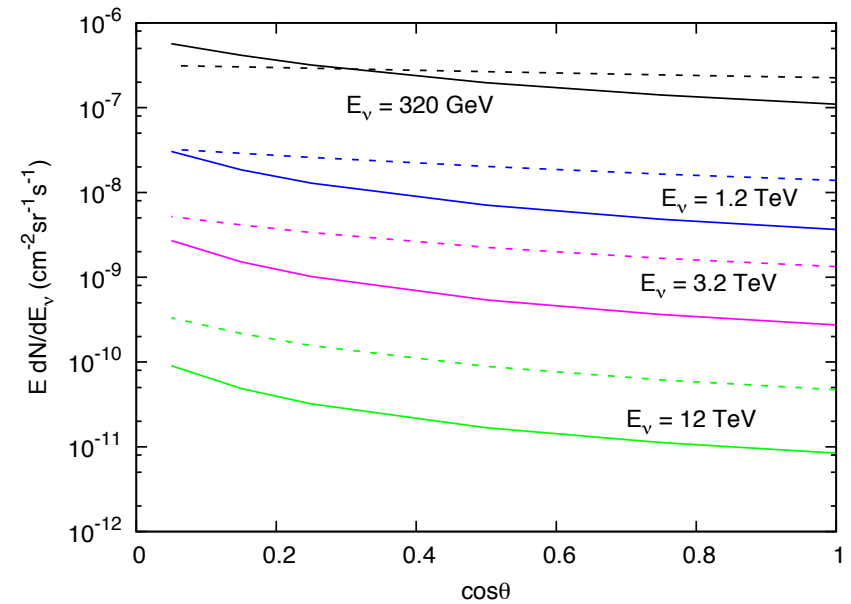
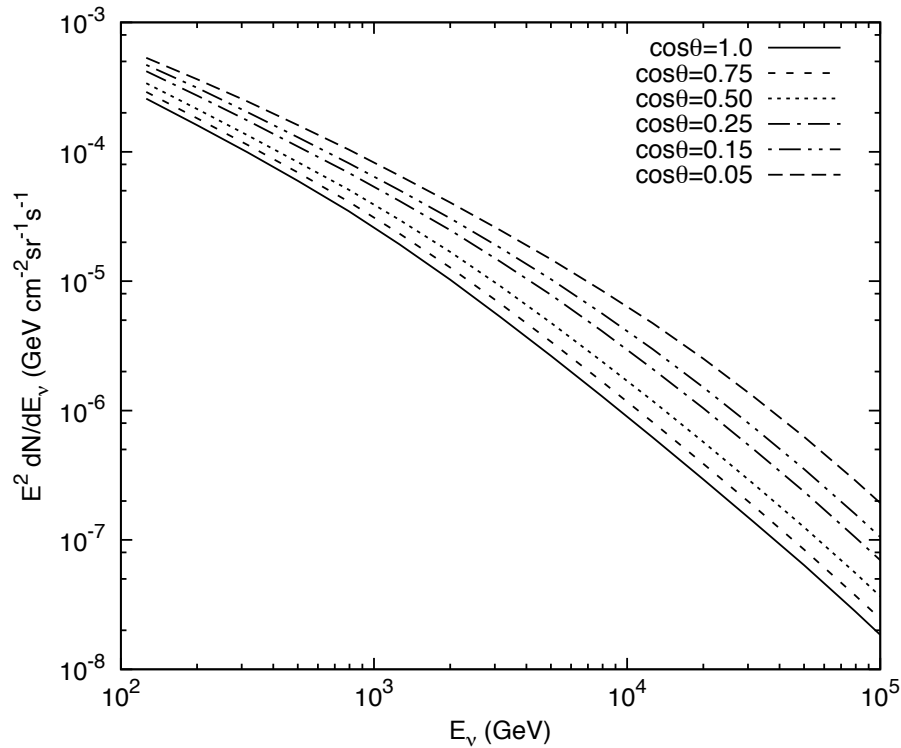
$$\frac{dN_\ell}{dE_\ell} = \frac{N_0(E_\ell)}{1 - Z_{NN}} \left\{ \frac{A_{\pi\ell}}{1 + B_{\pi\ell} \cos \theta E_\ell / \epsilon_\pi} + \frac{A_{K\ell}}{1 + B_{K\ell} \cos \theta E_\ell / \epsilon_K} \right\}$$

- Steepening first for π^\pm component
 - Later for kaon component ($\epsilon_\pi \ll \epsilon_K$)
- Steepening first for near vertical
 - Later for more horizontal ($1/\cos\theta$)
- π to ν_μ suppressed
 - Muon carries most of energy in $\pi \rightarrow \mu + \nu_\mu$
- Kaon channel dominates for $E(\nu_\mu) > 100$ GeV

Evolution of angular distribution

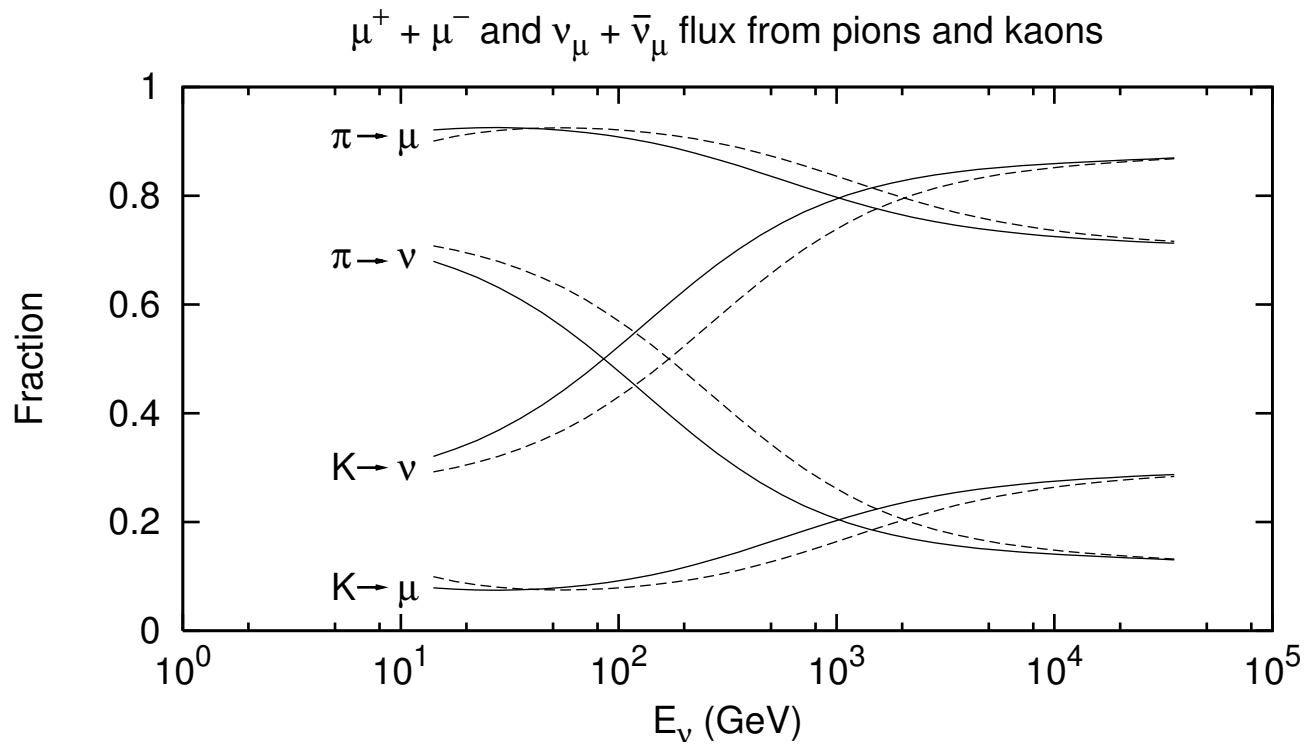
$$\nu_{\mu} + \bar{\nu}_{\mu}$$

π —
K - - -



For $E \approx$ TeV, pion component $\sim \sec(\theta)$, but kaon still isotropic

Pion, kaon fractions vs. E for ν_μ & μ



Summary of μ^+/μ^- measurements

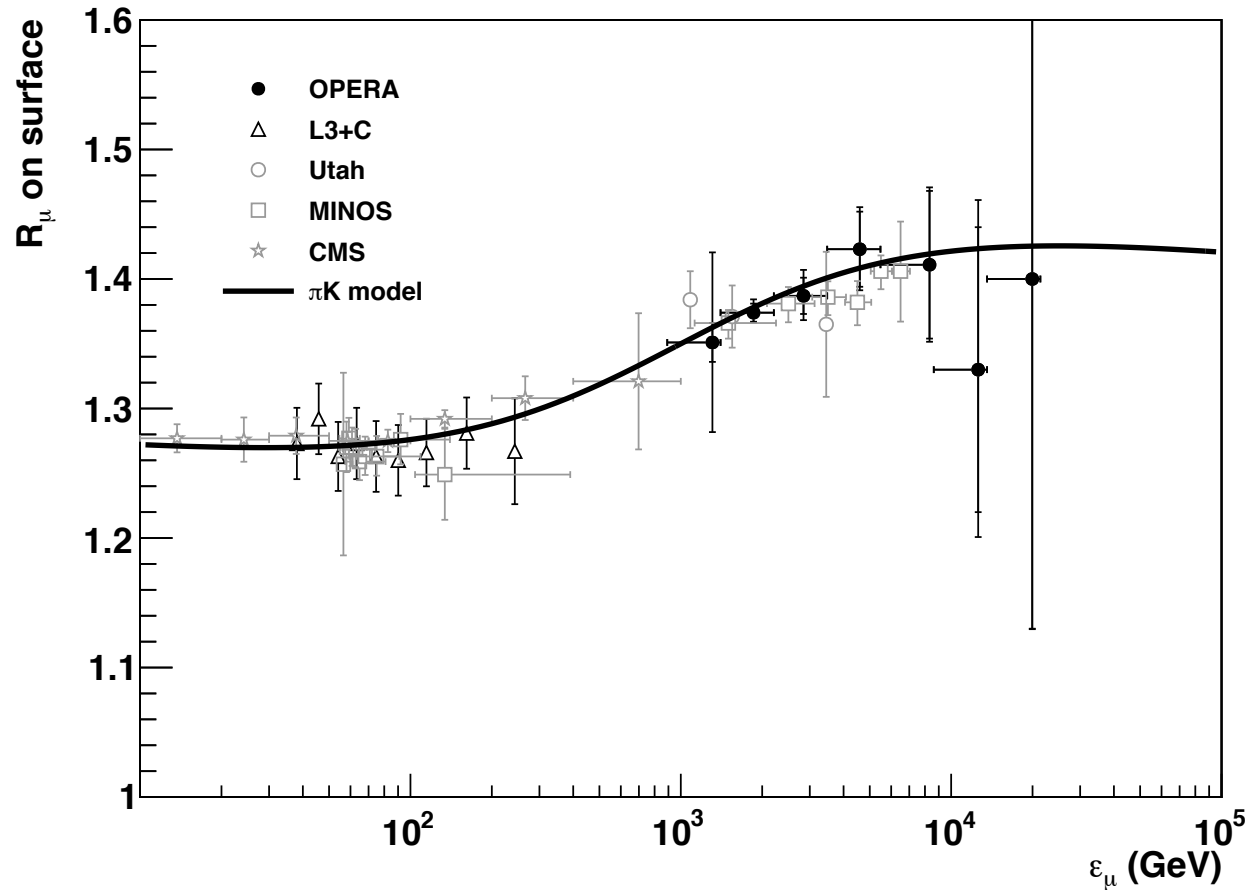


Figure from OPERA, N. Agafonova et al., Eur. Phys. J C74 (2014) 2933

Muon charge ratio

Pions only (Frazer et al., PR D 5 (1972) 1653)

$$\frac{\mu^+}{\mu^-} \approx \frac{1 + \beta\delta_0\alpha_\pi}{1 - \beta\delta_0\alpha_\pi} = \frac{f_{\pi^+}}{1 - f_{\pi^+}},$$

$$\beta = \frac{1 - Z_{pp} - Z_{pn}}{1 - Z_{pp} + Z_{pn}} \approx 0.909;$$

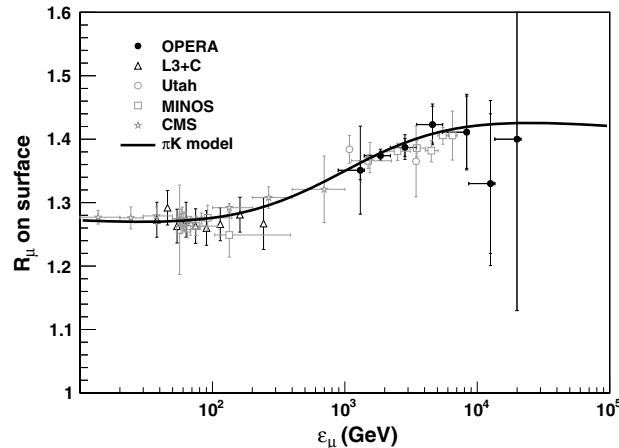
$$\alpha_\pi = \frac{Z_{p\pi^+} - Z_{p\pi^-}}{Z_{v\pi^+} + Z_{v\pi^-}} \approx 0.165$$

Include $K \rightarrow \mu + \nu_\mu$

TG Astropart. Phys. 35(2012) 801

$$\frac{\mu^+}{\mu^-} = \left[\frac{f_{\pi^+}}{1 + B_{\pi\mu} \cos(\theta) E_\mu / \epsilon_\pi} + \frac{\frac{1}{2}(1 + \alpha_K \beta \delta_0) A_{K\mu} / A_{\pi\mu}}{1 + B_{K\mu}^+ \cos(\theta) E_\mu / \epsilon_K} \right] \times \left[\frac{(1 - f_{\pi^+})}{1 + B_{\pi\mu} \cos(\theta) E_\mu / \epsilon_\pi} + \frac{(Z_{NK^-} / Z_{NK}) A_{K\mu} / A_{\pi\mu}}{1 + B_{K\mu} \cos(\theta) E_\mu / \epsilon_K} \right]^{-1}$$

$$\alpha_K = \frac{Z_{pK^+} - Z_{pK^-}}{Z_{pK^+} + Z_{pK^-}}$$



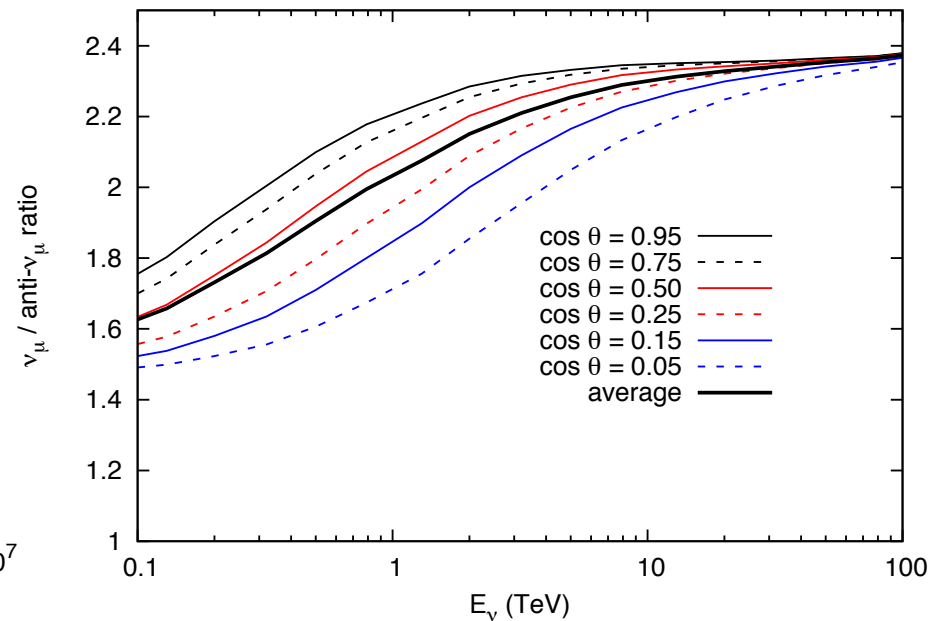
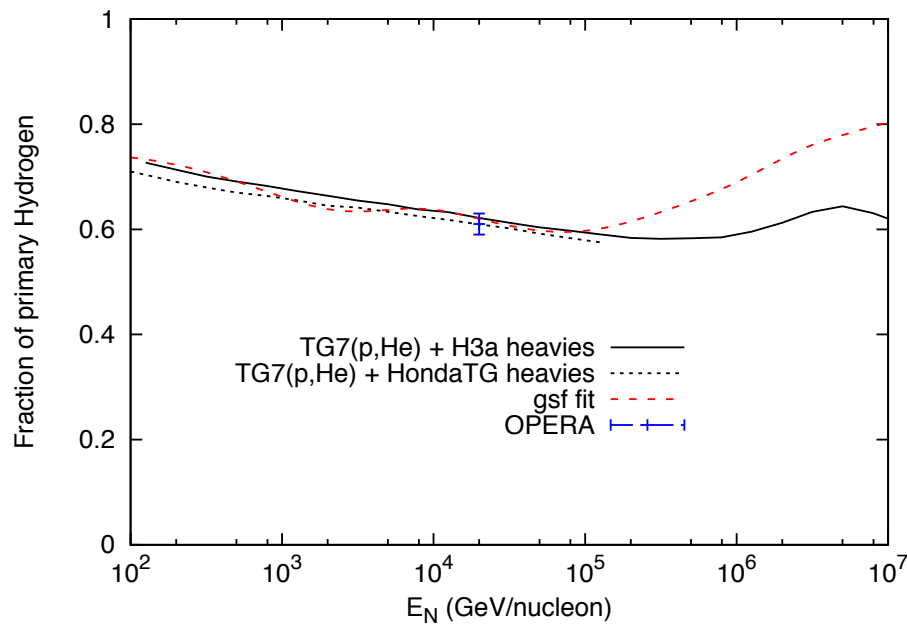
Rise in muon charge ratio reflects higher asymmetry in the charged kaon channel, which becomes more important when $E_\mu > \epsilon_K \approx 850$ GeV.

The key parameter is $\alpha_K > \alpha_\pi$ due to associated production: $p \rightarrow \Lambda K^+$

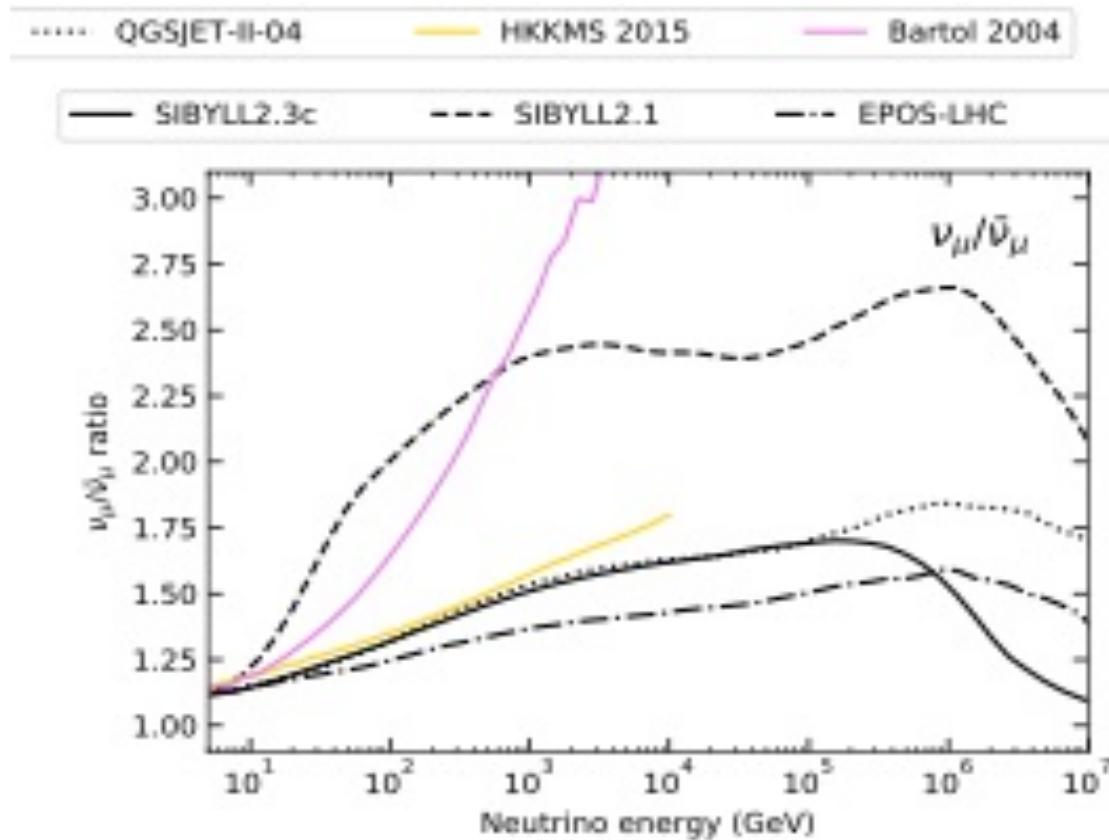
Use OPERA results to predict $\nu_\mu/\bar{\nu}_\mu$

$$\delta_0 = \frac{p - n}{p + n} = 0.61$$

$$\alpha_K = \frac{Z_{pK^+} - Z_{pK^-}}{Z_{pK^+} + Z_{pK^-}} = 0.51$$

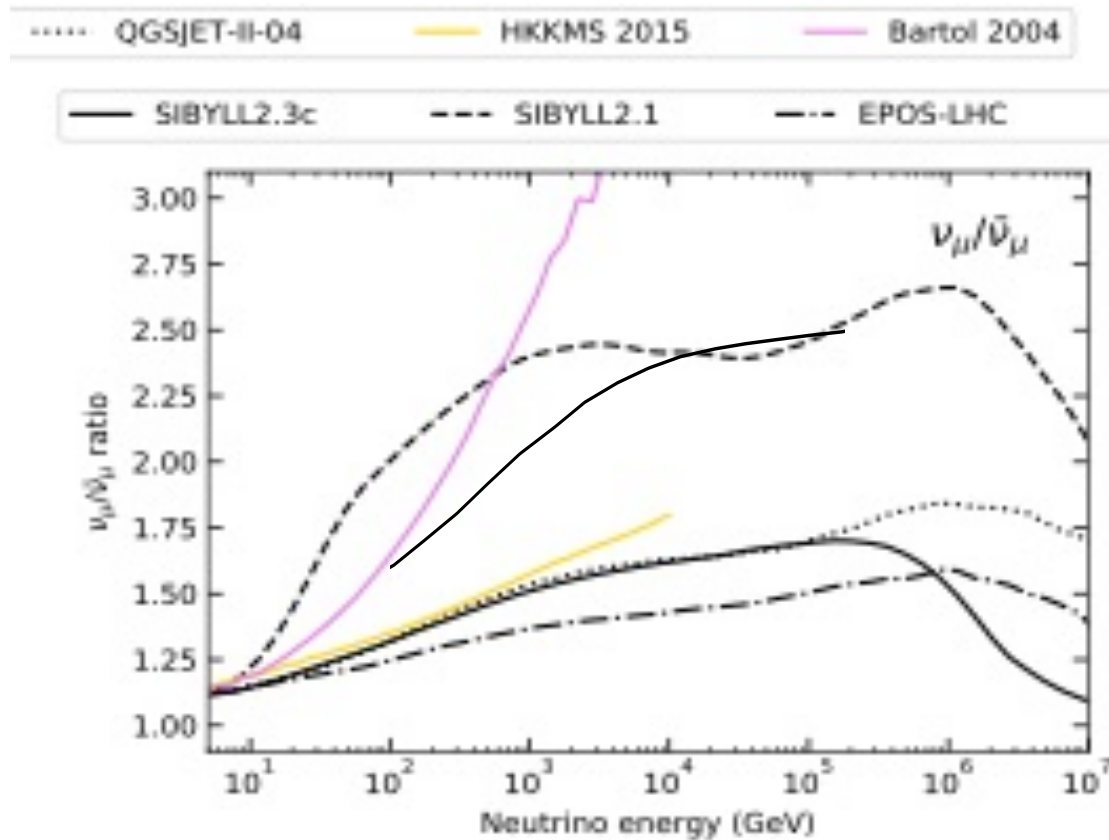


Muon neutrino/anti-neutrino ratio predicted in 6 hadronic interaction models



From Fedynitch et al., PRD 100 103018 (2019)

Muon neutrino/anti-neutrino ratio predicted in 6 hadronic interaction models



IceCube sterile neutrino analysis

PRL 125, 141801 (2020)

Matter-enhanced resonance for
near vertically upward anti- $\bar{\nu}_\mu$

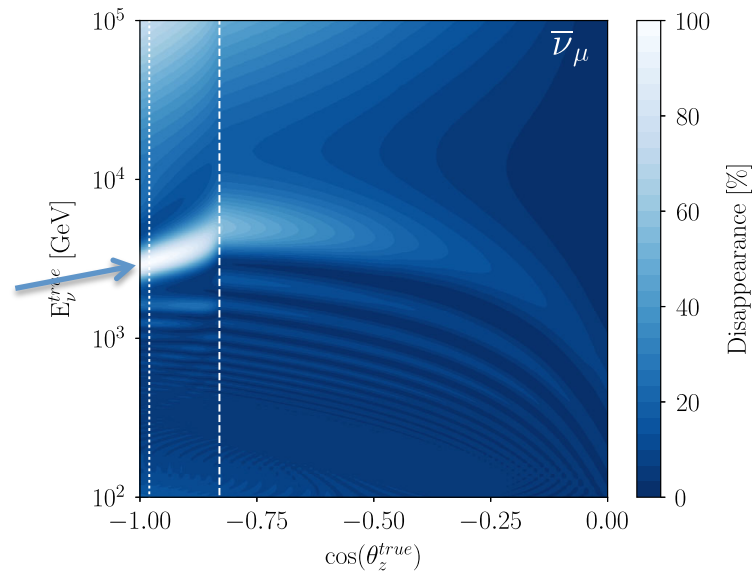
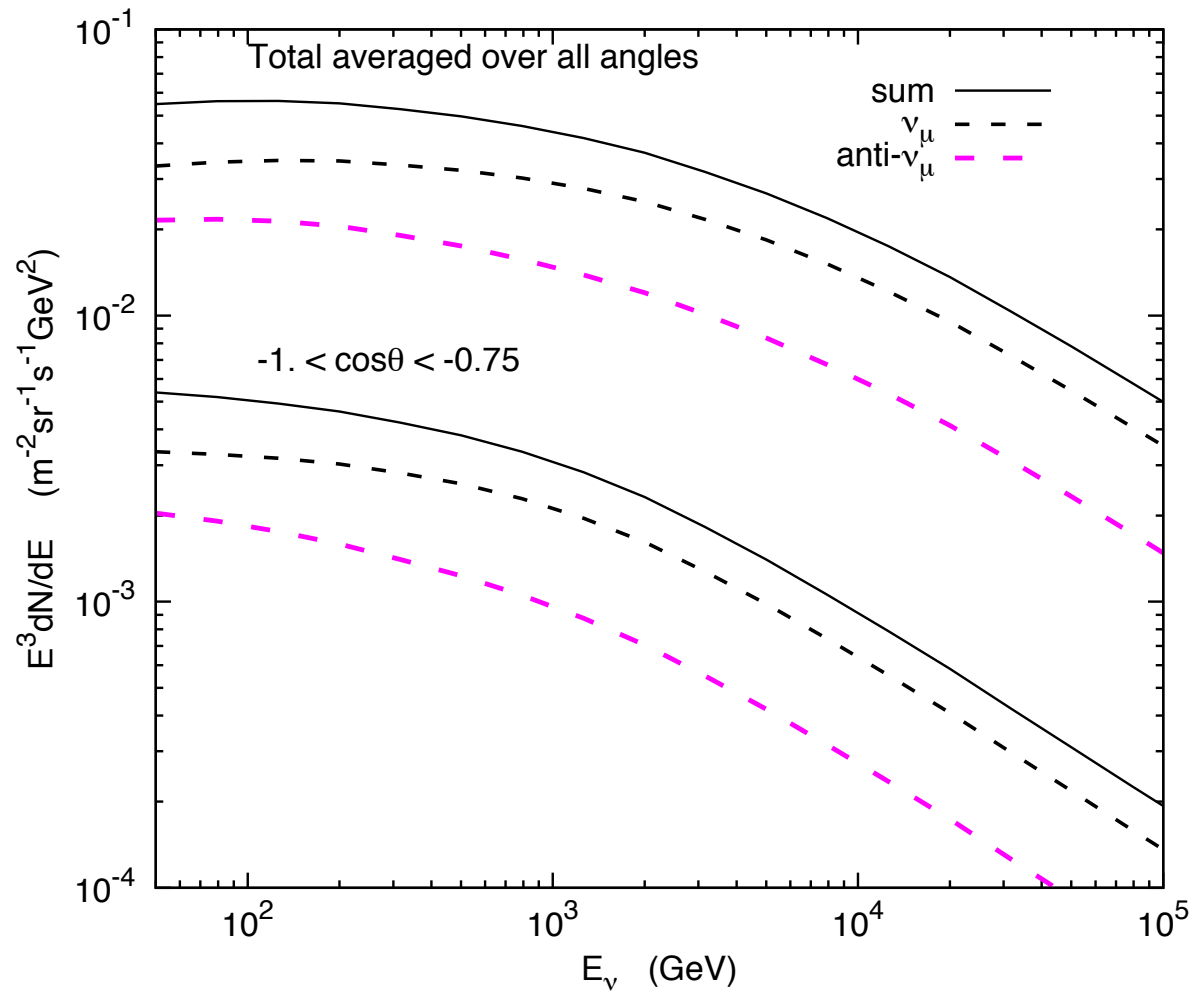


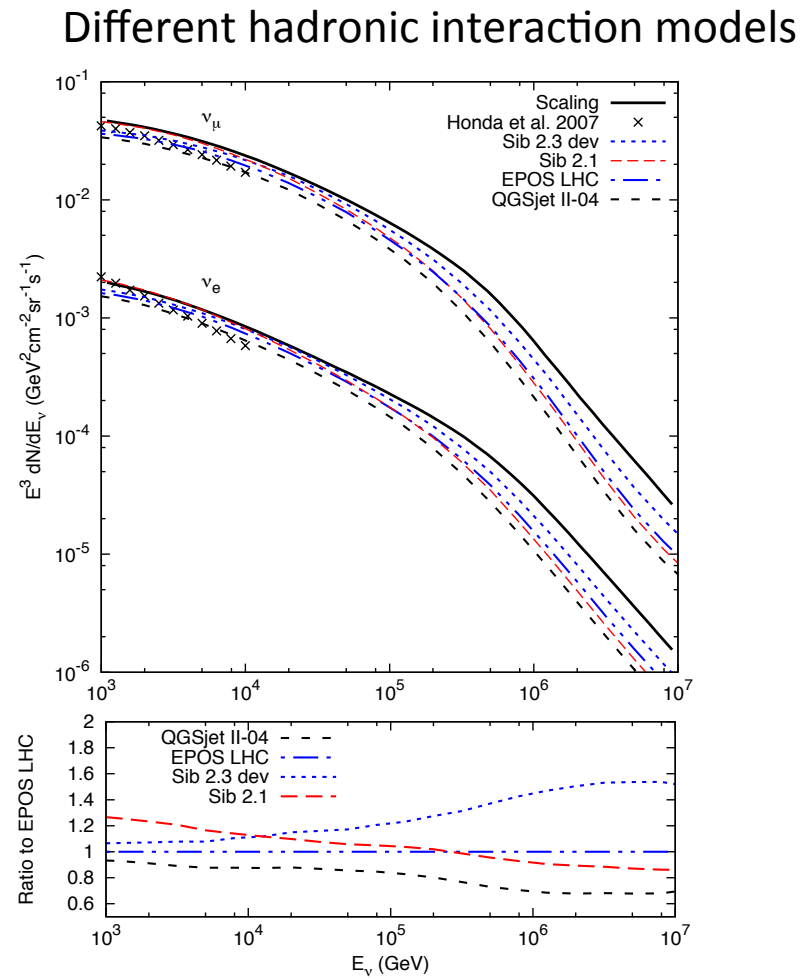
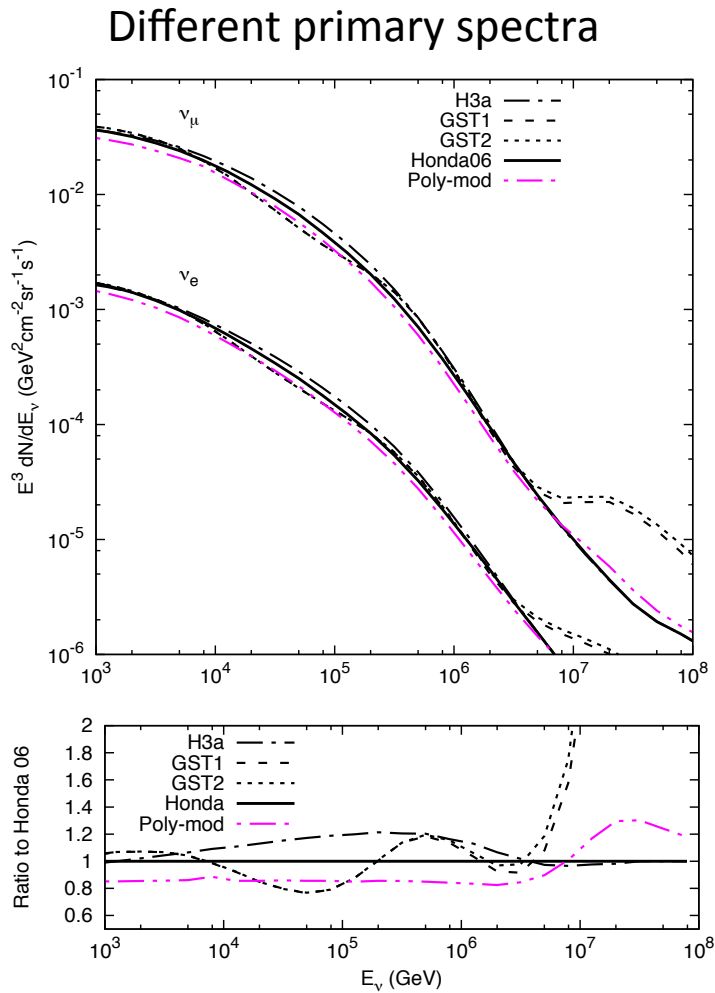
FIG. 1. Muon-antineutrino oscillogram. Atmospheric $\bar{\nu}_\mu$ disappearance probability vs true energy and cosine zenith at the globally preferred sterile neutrino hypothesis of Ref. [11] [$\Delta m_{41}^2 = 1.3 \text{ eV}^2$, $\sin^2(2\theta_{24}) = 0.07$, $\sin^2(2\theta_{34}) = 0.0$]. Effects include a matter-enhanced resonance at TeV energies, neutrino absorption at high energy and small zenith, and vacuumlike oscillation at low energies. The matter-enhanced resonance appears only in the antineutrino flux for the case of small angles and $\Delta m_{41}^2 > 0$. Vertical white lines indicate transitions between inner to outer core [$\cos(\theta_z^{true}) = -0.98$] and outer core to mantle [$\cos(\theta_z^{true}) = -0.83$].

Separate ν_μ & $\bar{\nu}_\mu$ fluxes



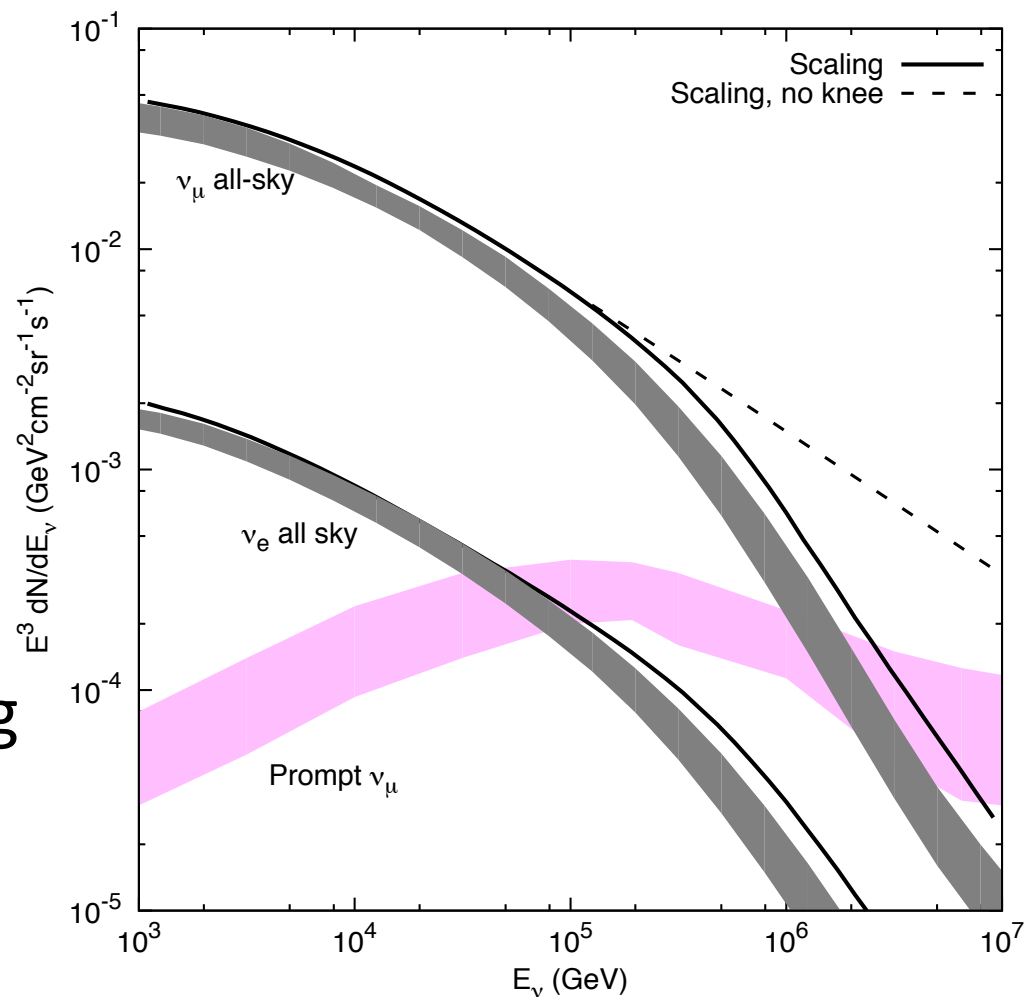
Uncertainties in conventional ν fluxes:

TG: 1605.03073



Summary:

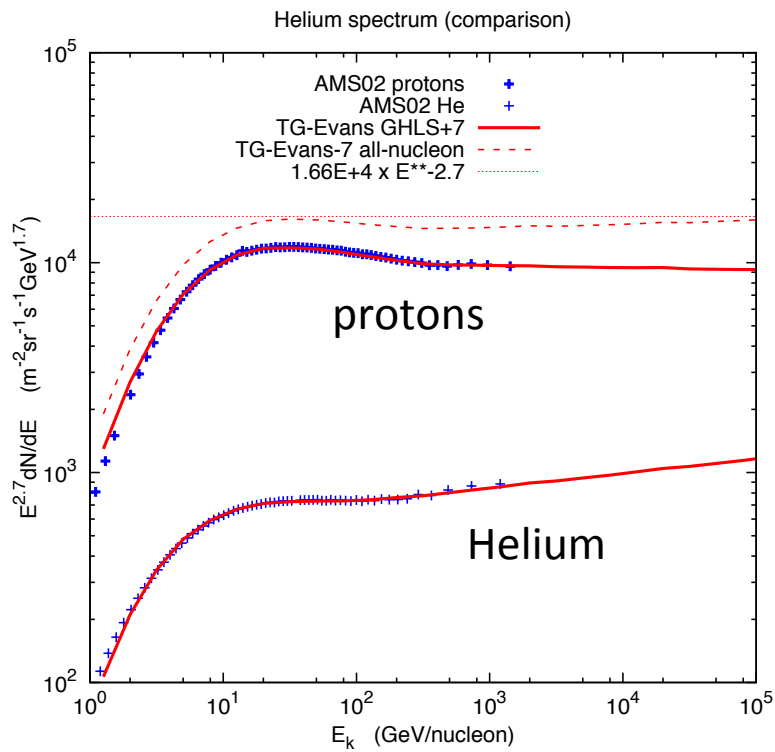
- Hadronic models
 - Gentle non-scaling
- Energy spectrum
 - Gradual steepening
 - Not a sharp break
- Prompt
 - Negligible for ν_μ below 100 TeV



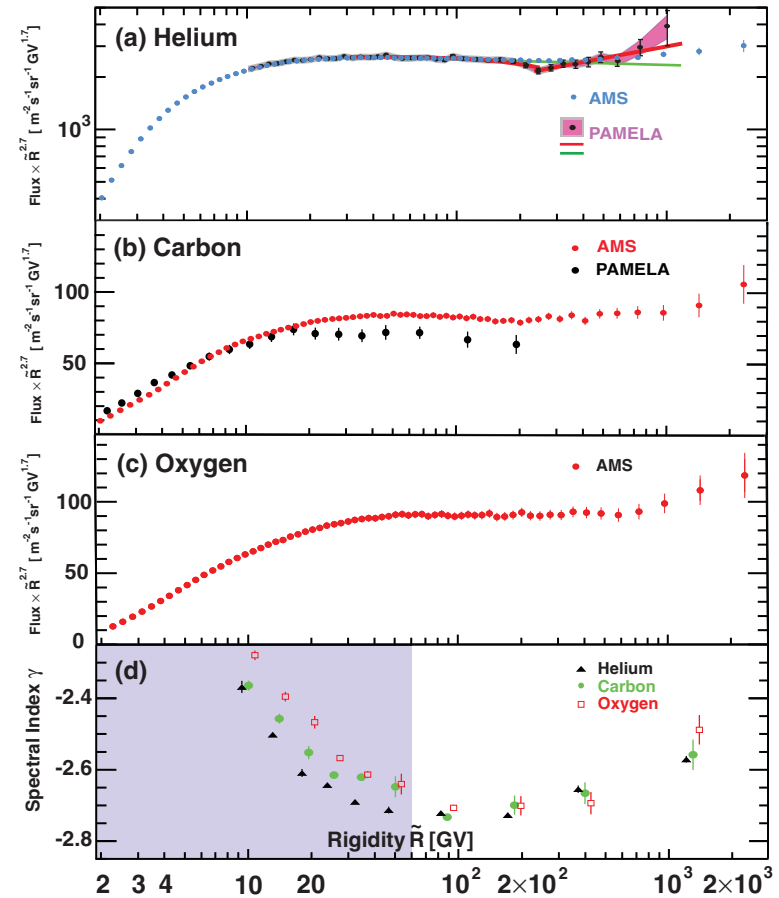
Shaded bands for conventional span
Sib2.3, Epos LHC, QGSjet II-04,
prompt: PROSA, Sib2.3, GRRST,
BERSS

Backup on primary spectrum and composition

AMS 02

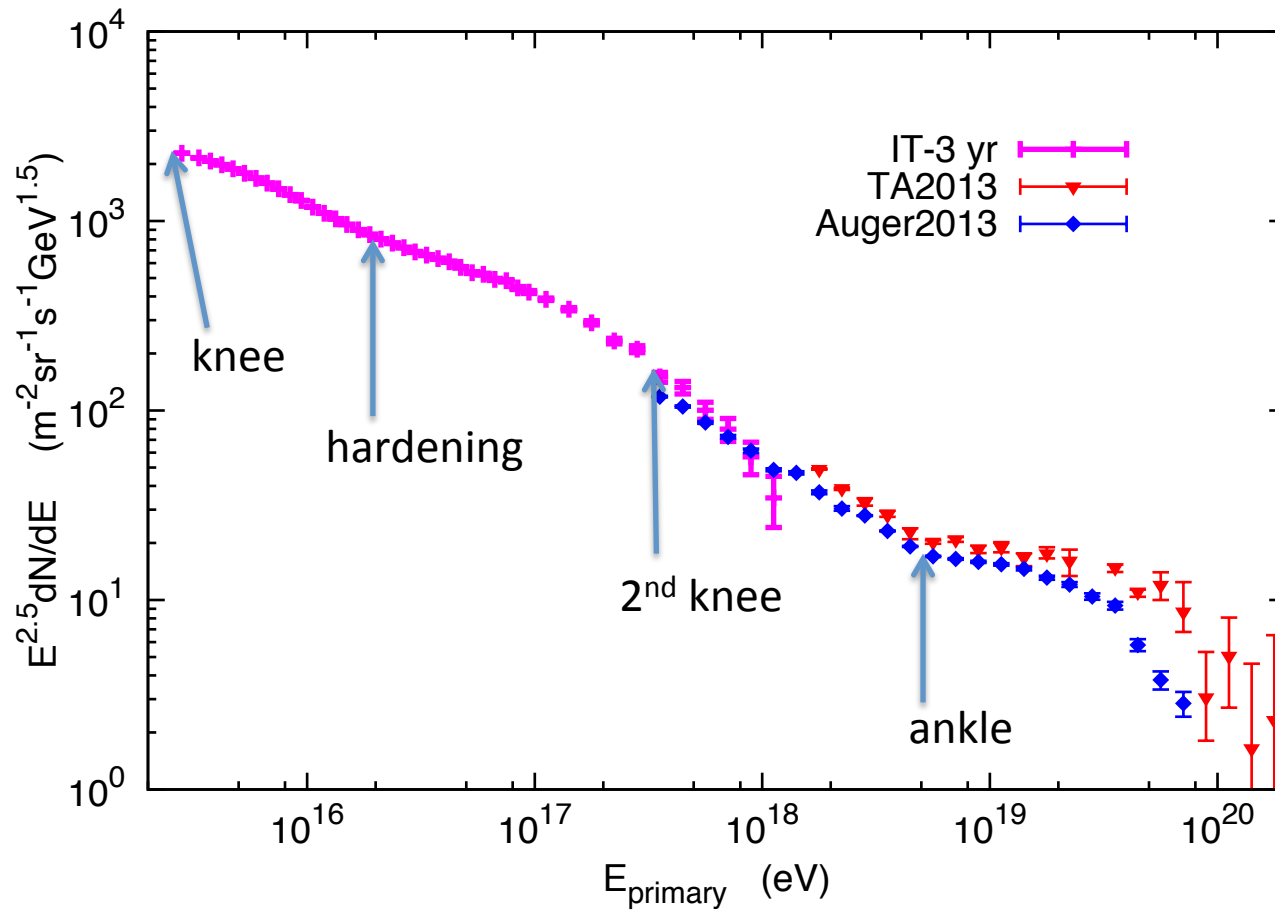


Rigidity dependent hardening



AMS02, PRL 119 (2017) 251101

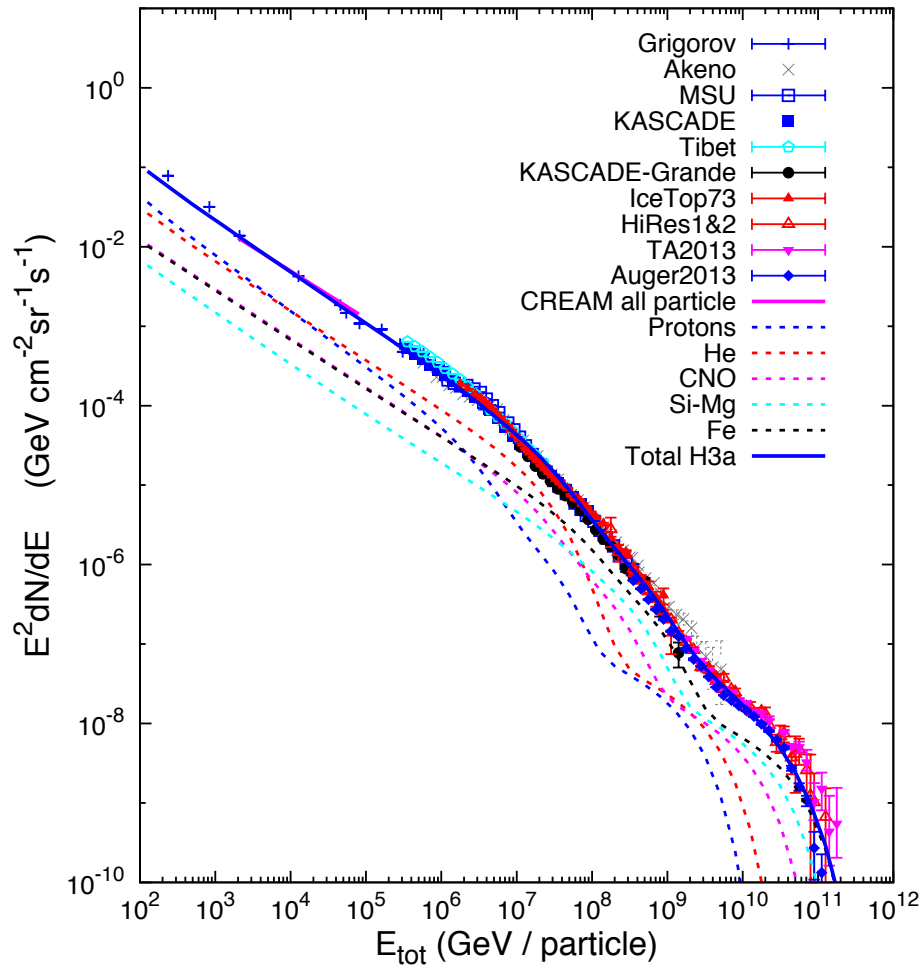
Features in all-particle spectrum



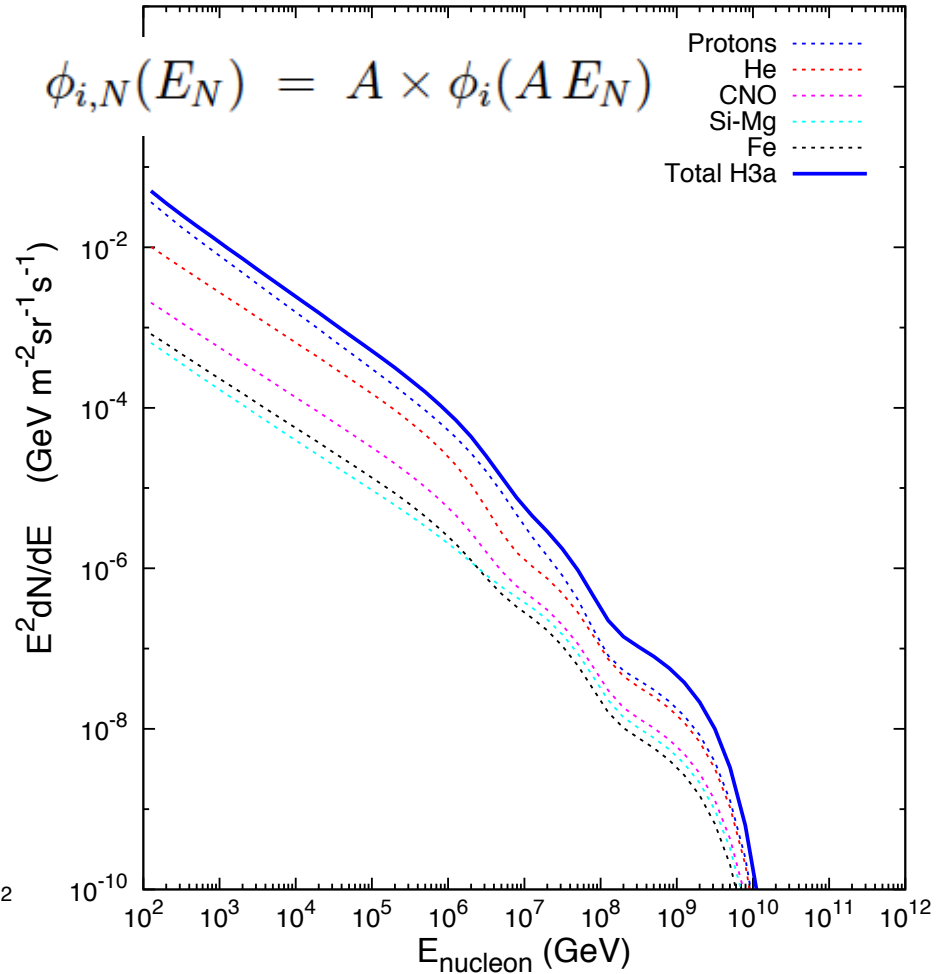
All-particle spectrum to nucleon spectrum

$$\phi_i(E) \equiv E \frac{dN_i}{dE} = \sum_{j=1}^3 a_{i,j} E^{-\gamma_{i,j}} \times \exp \left[-\frac{E}{Z_i R_{c,j}} \right]$$

All-particle spectrum



Spectrum of nucleons



Three-population models

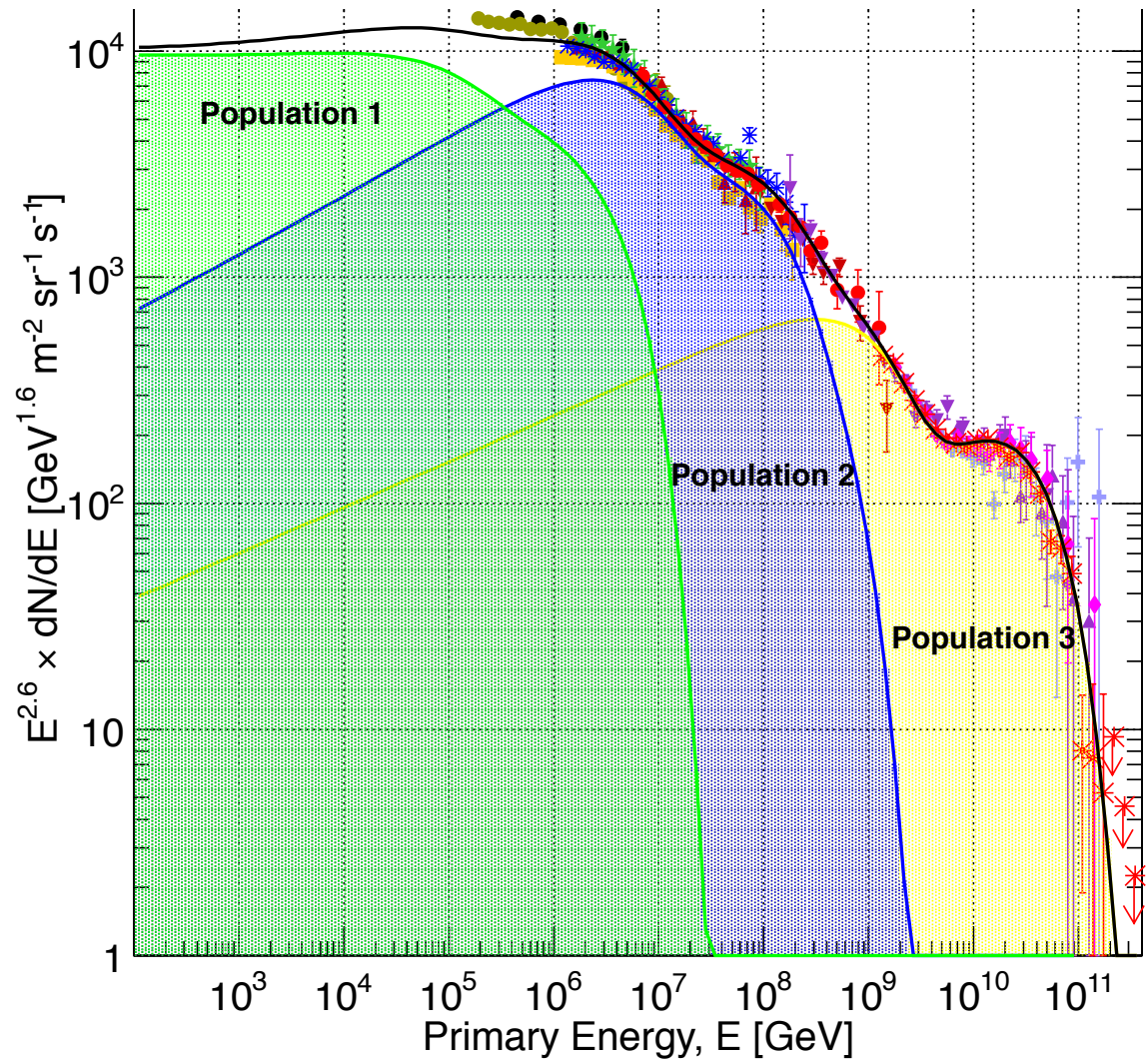
		p	He	CNO	Mg-Si	Fe
Galactic A	Pop. 1:	7860	3550	2200	1430	2120
	$R_c = 4$ PV	1.66	1.58	1.63	1.67	1.63
Galactic B	Pop. 2:	20	20	13.4	13.4	13.4
	$R_c = 30$ PV	1.4	1.4	1.4	1.4	1.4
Extragalactic { H3a →	Pop. 3:	1.7	1.7	1.14	1.14	1.14
	$R_c = 2$ EV	1.4	1.4	1.4	1.4	1.4
H4a →	Pop. 3(*):	200	0.0	0.0	0.0	0.0
	$R_c = 60$ EV	1.6				

TG Astropart. Phys. 35 (2012) 801

		p	He	C	O	Fe	$50 < Z < 56$	$78 < Z < 82$
GST	Pop. 1:	7000	3200	100	130	60		
	$R_c = 120$ TV	1.66	1.58	1.4	1.4	1.3		
	Pop. 2:	150	65	6	7	2.3	0.1	0.4
	$R_c = 4$ PV	1.4	1.3	1.3	1.3	1.2	1.2	1.2
Pop. 3:	14				0.025			
$R_c = 1.3$ EV	1.4				1.2			

TG, Stanev, Tilav, Front. Phys. (Beijing) 8 (2013) 748 (arXiv:1303.3565)

GST 3 population model



GSF (Global Spline Fit)

“Data-driven”, no input model (H. Dembinski et al., 1711.11432)

