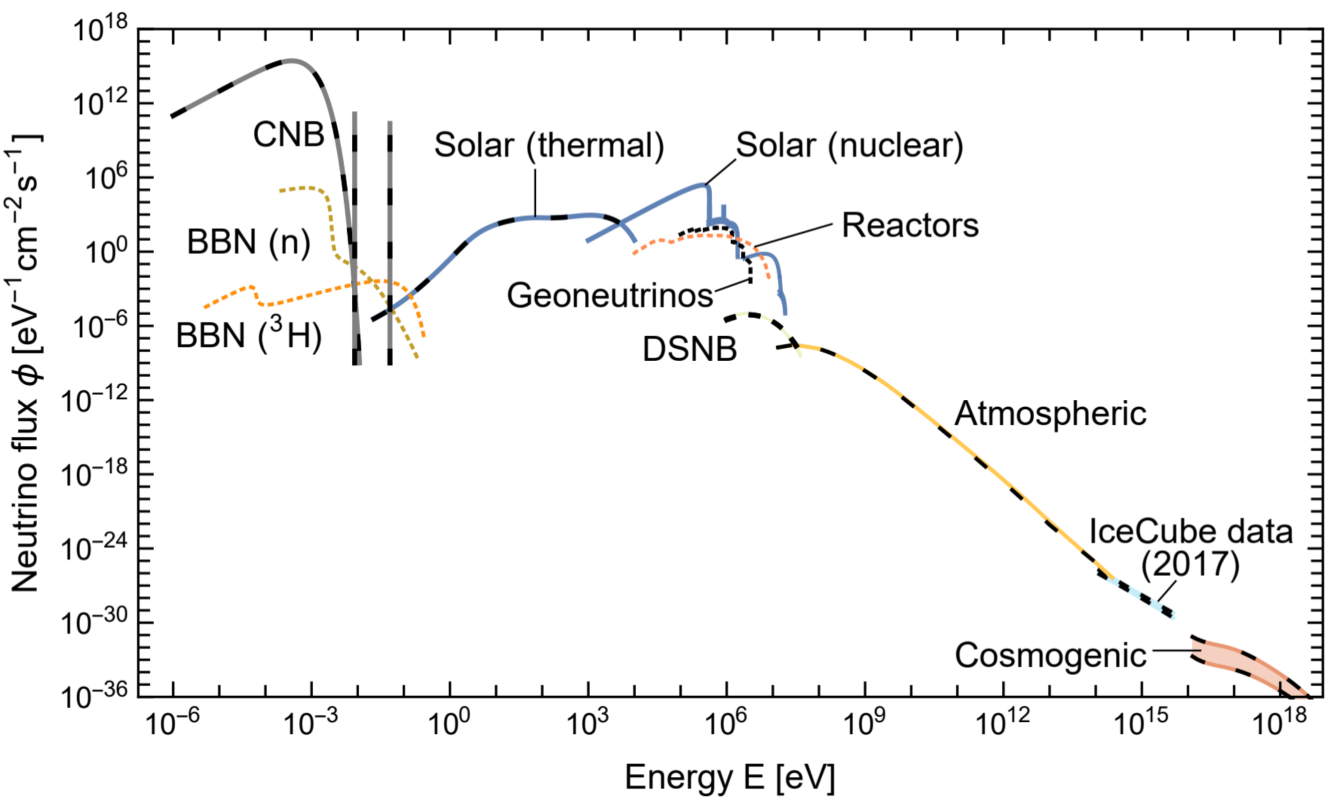
Neutrino Interactions, Simulation, and Event Generation

Lecture 1: Neutrino interactions in Matter

The Grand Unified Neutrino Spectrum



Vitagliano, Tamborra, Raffelt 2019

"Flux" = how many neutrinos pass through a given unit area in a given unit time (at a given unit energy)?

$$\phi \left[\frac{\nu}{cm^2 \, eV \, s} \right]$$

How do we translate this to a rate?

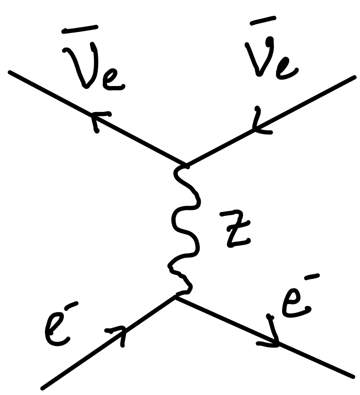
$$N = \phi \times \sigma \times N_{targets} \times \varepsilon$$

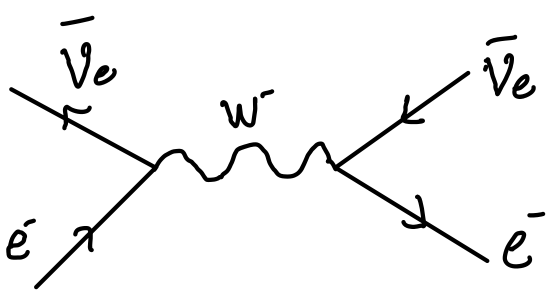
rate — flux — cross section — number of targets — detection efficiency

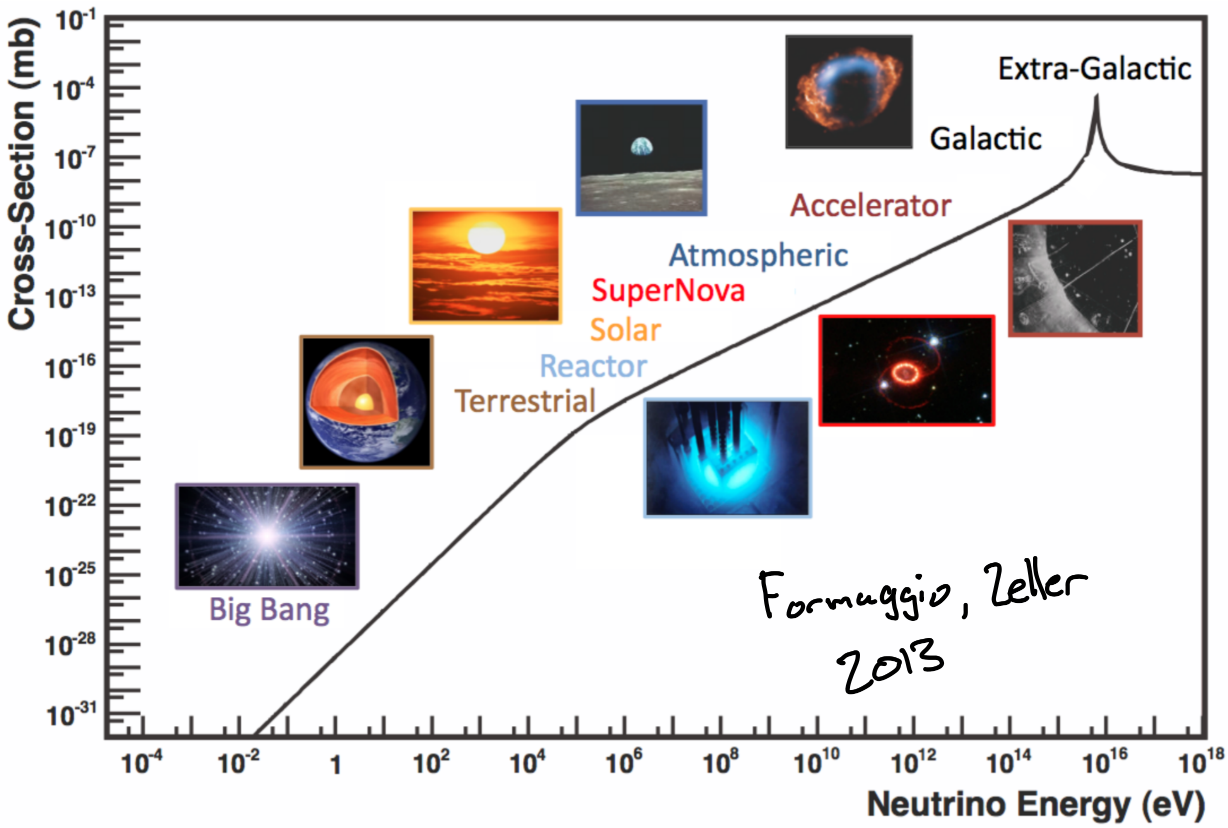
"cross section" ≈ how likely is an interaction?

In general, $\sigma = \sigma(E_\nu)$

Take $\bar{\nu}_e + e^- \to \bar{\nu}_e + e^-$





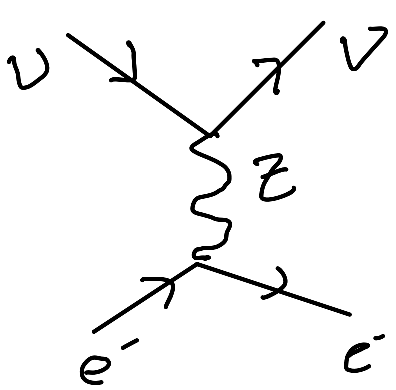


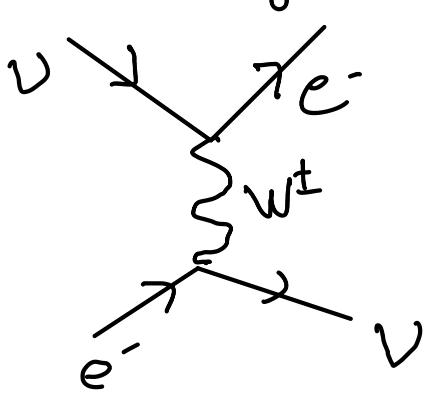
(note : m = "millibarn" = $10^{-27}\ cm^2$)

Now let's consider neutrino - electron scattering

$$\nu_e + e^- \rightarrow \nu_e + e^-$$

two contributions : Neutral Current (NC) and Charged Current (CC)





Following Mohapatra, Pal Ch 2.2-2.3

$$\frac{d\sigma}{dy}(\nu_e e^- \to \nu_e e^-) \approx \frac{2G_F^2 m_e E_\nu}{\pi}\left[g_L^2 + g_R^2(1-y)^2\right]$$

where

$$G_F = \frac{\sqrt{2}}{8}\frac{g_w^2}{M_W^2} \approx 5.29 \times 10^{-38}\ \frac{cm^2}{GeV^2} \quad \text{(Fermi Constant)}$$

$$m_e = 511\ \text{keV (electron mass)}$$

$$g_L = \begin{cases} \frac{1}{2} + \sin^2\theta_W & \nu_\ell = \nu_e \\ -\frac{1}{2} + \sin^2\theta_W & \nu_\ell = \nu_\mu, \nu_\tau \end{cases}$$

$$g_R = \sin^2\theta_W$$

electron couplings to W/Z bosons

$$\sin^2\theta_W \approx 0.23 \quad \text{(weinberg angle)}$$

and most importantly, the <u>inelasticity</u>:

$y = \frac{T}{E_\nu}$, where $T \equiv E_{e^-} - m_e$ is the energy transferred to the electron

$$\sigma(\nu_e e^- \to \nu_e e^-) = \int_0^1 \frac{d\sigma}{dy}dy \approx \frac{2G_F^2 m_e E_\nu}{\pi}\left(g_L^2 + \frac{1}{3}g_R^2\right)$$

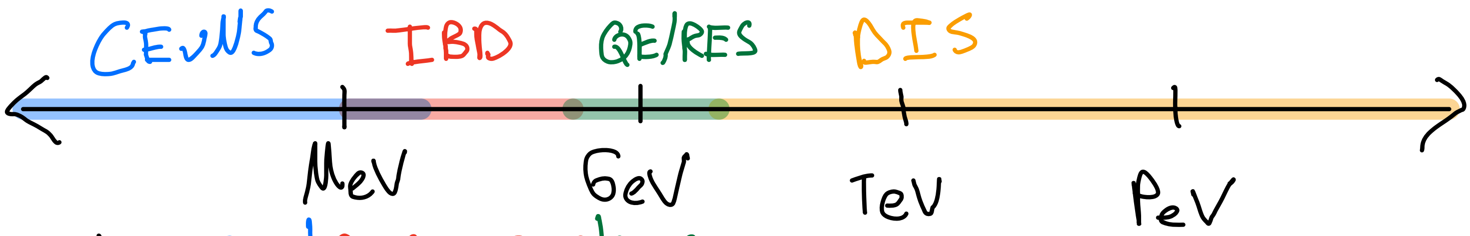
for $\bar{\nu}_e e^- \to \bar{\nu}_e e^-$, take $g_L \leftrightarrow g_R$

plugging in numbers:

$$\sigma\{\nu_e(\bar{\nu}_e)e^-\} \approx 0.9\,(0.38) \times 10^{-43}\left(\frac{E_\nu}{10\ \text{MeV}}\right)\ cm^2$$

Neutrinos mostly interact with nuclei / nucleons in experiments. Different things happen at different energy ranges.

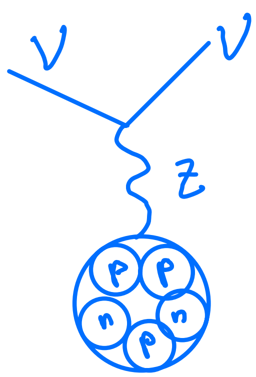
Following Formaggio, Zeller 2013



target: nucleus nucleons quarks

Coherent Elastic Neutrino Nucleus Scattering

$E_\nu \lesssim 1\ \mathrm{MeV}$



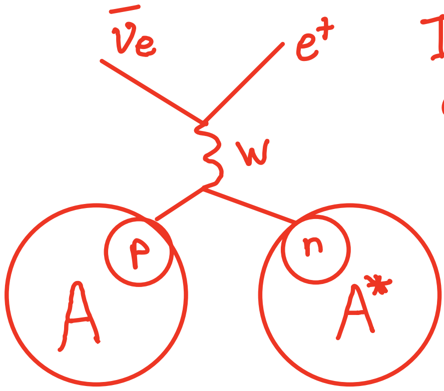
Coherent enhancement:

$$Q \ll R^{-1} \Rightarrow \sigma \propto A^2$$

momentum transfer — nuclear radius — atomic number

low recoil energy: $T_{max} = \dfrac{E_\nu}{1 + \frac{M_A}{2E_\nu}}$

Inverse Beta Decay $1.806 < \frac{E_\nu}{\mathrm{MeV}} \lesssim 10$



Intimately connected to nuclear beta decay — one predicts the other

Distinct coincidence signature:

e^+ annihilation with e^-, n capture on nucleus followed by emission of monoenergetic photon

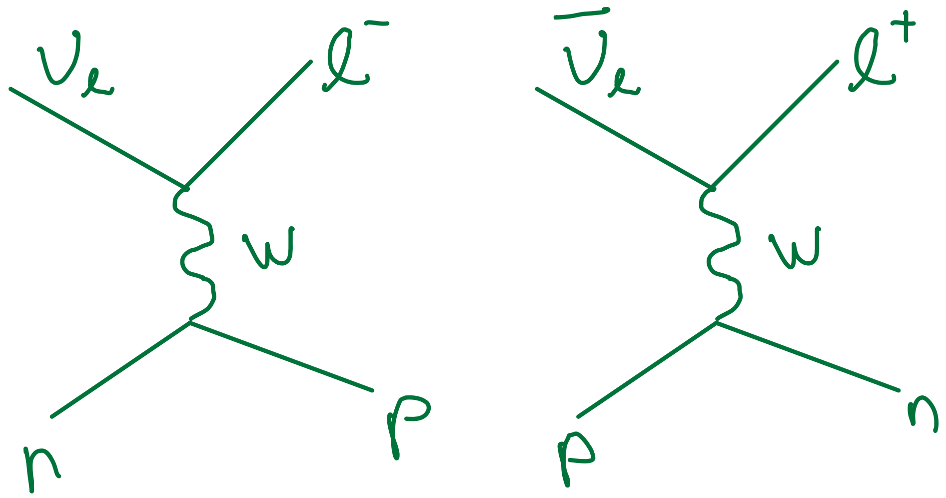
This is the powerhouse of neutrino physics

- First neutrino detected: IBD of reactor neutrino (Cowan + Reines 1956)
- First solar neutrino detection: IBD with neutrinos, $\nu_e + {}^{37}Cl \to {}^{37}Ar + e^-$ (Davis, Bachall 1976)

(Quasi-) elastic and Resonance Production

$0.1 \lesssim \frac{E_\nu}{GeV} \lesssim 20$

QE:



neutrino scatters elastically off of an individual nucleon, kicking it out of the nucleus

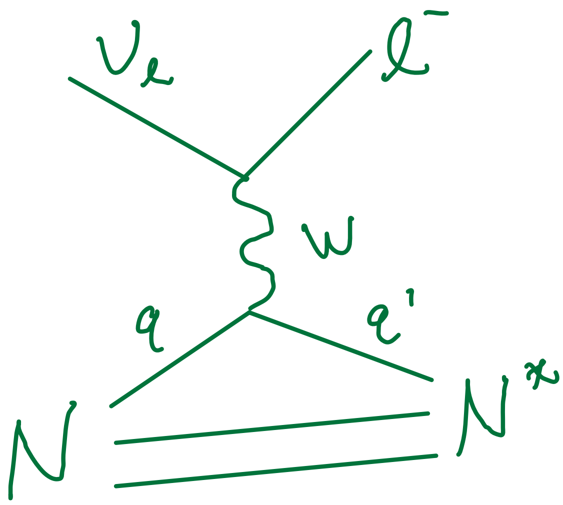
roughly speaking $\frac{d\sigma}{dQ^2} \propto \frac{G_F^2}{E_\nu^2} \underbrace{F(q^2)}_{\text{nucleon form factors,}}$ ← momentum transfer, $(P_\nu - P_l)^2$

nucleon form factors,
vector $F_{1,2}$
axial vector F_A
pseudoscalar F_P

nuclear environment matters!

- nucleons are not at rest
- they undergo final state interactions (FSI)
- they can be correlated with other nucleons, including meson exchange currents (MEC)

RES:



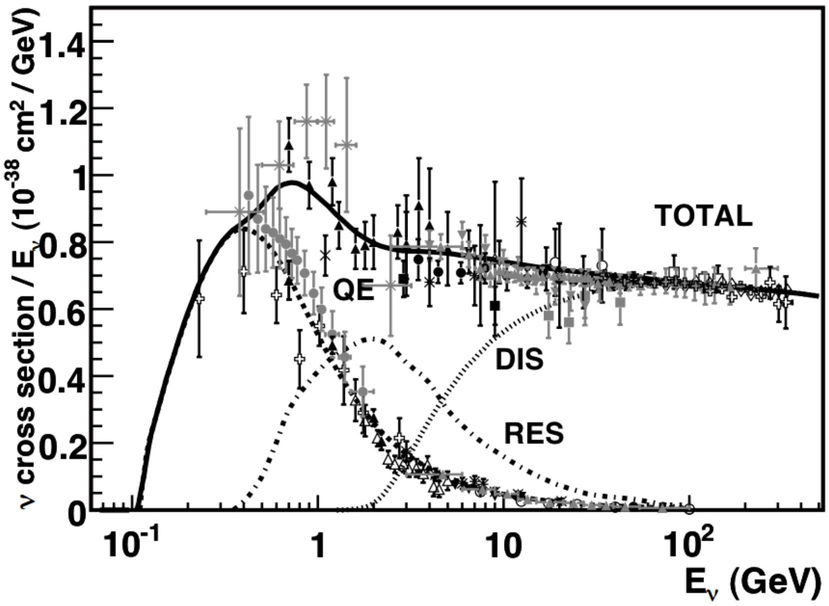
$\rightarrow$ baryon resonance e.g. $\Delta(1232)$

nucleons have 3 valence quarks, which can transition in charged-current interactions

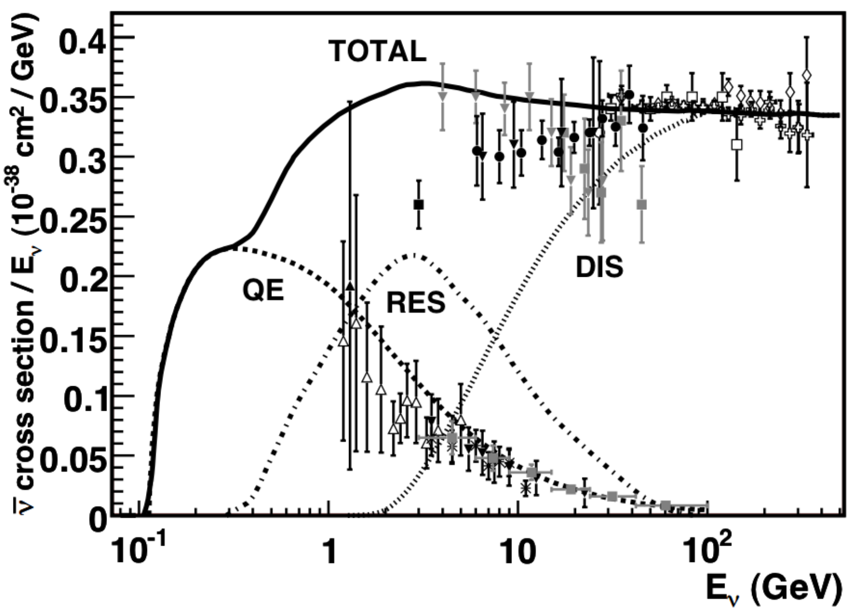
most common case: $N^* \rightarrow N\pi$, single pion production

less common cases: coherent pion production,
multi pion production
Kaon production

this is a <u>messy</u> region! As we transition from QE to DIS, the hadronic system becomes more complicated. Many historical disagreements between theory and experiment (if interested, look into the "axial mass anomaly")



Formaggio, Zeller 2013

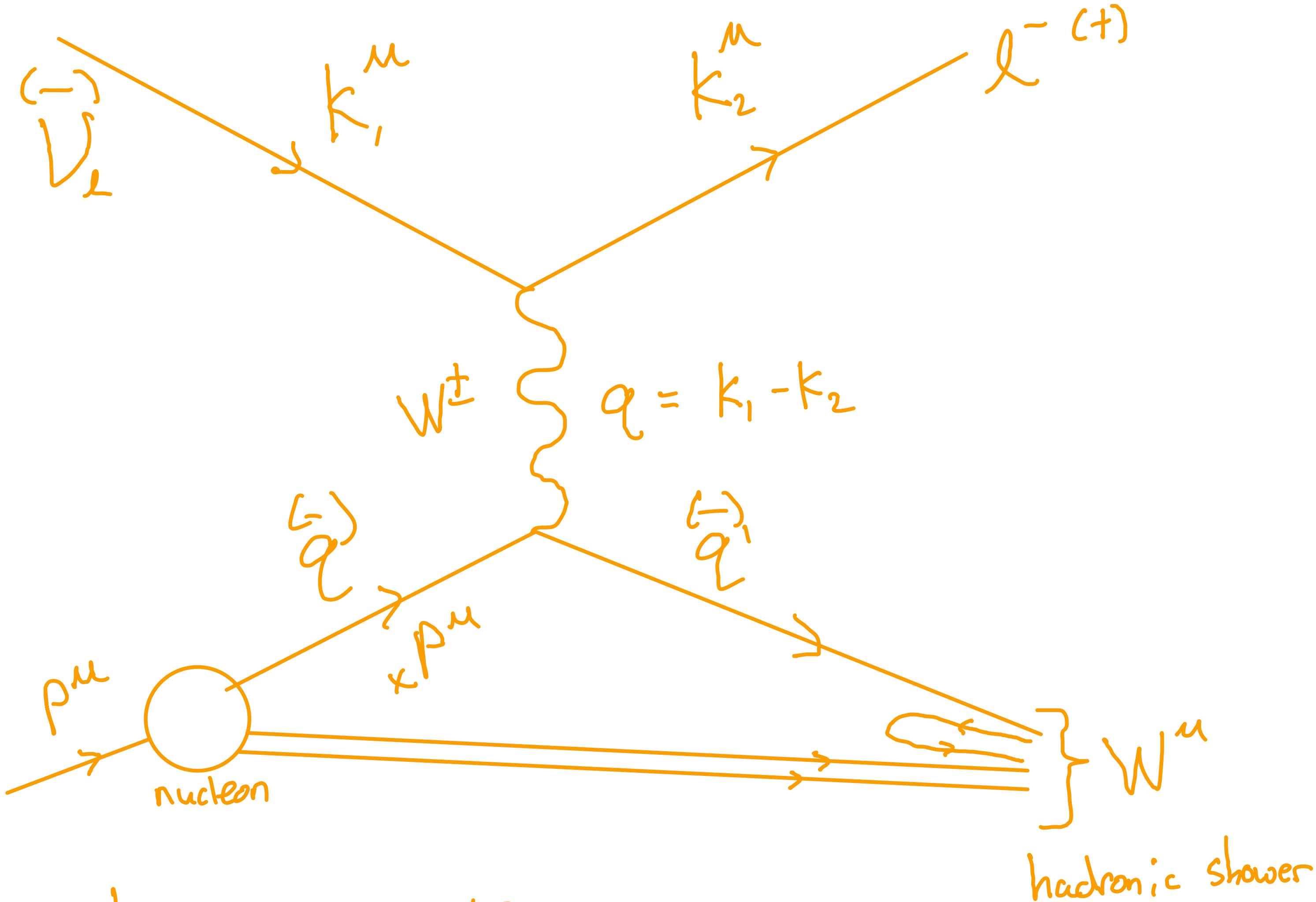


note lack of $\bar{\nu}$ data at low energies!

Deep Inelastic Scattering $E_\nu \gtrsim 20$ GeV

neutrinos scatter off of quarks when their energy is high enough to probe such small length scales

This is the most important regime for astro neutrino experiments, so we'll dive deeper here



observables: E_ℓ, E_{had}, θ_ℓ

Looks complicated, right? Actually pretty well-understood process in perturbative QCD.

Three important Lorentz-invariant quantities

inelasticity $\quad y \equiv \dfrac{p \cdot q}{p \cdot k_1} \overset{lab}{=} \dfrac{E_{hadrons}}{E_\nu}$

momentum transfer $\quad Q^2 \equiv -q^2 \overset{lab}{=} -m_\ell^2 + 2E_\nu(E_\ell - p_\ell \cos\theta_\ell)$

Bjorken scaling variable $\quad x \equiv \dfrac{Q^2}{2p \cdot q} \overset{lab}{=} \dfrac{Q^2}{2M_N E_\nu y}$

Simplest picture for both CC and NC DIS

$$\frac{d^2\sigma^{\nu/\bar{\nu}}}{dx\,dy} = \frac{G_F^2 M_N E_\nu}{\pi\left(1+\frac{Q^2}{M_{W,Z}^2}\right)^2}\left[\frac{y^2}{2}2xF_1(x,Q^2) + \left(1-y-\frac{M_N x y}{2E_\nu}\right)F_2(x,Q^2) + /-\, y\left(1-\frac{y}{2}\right)xF_3(x,Q^2)\right]$$

CC v.s. NC

ν $\bar{\nu}$

$F_i(x,Q^2)$ are structure functions

that depend on <u>parton density functions</u>.

In the CC case (at leading order)

$$F_2(x,Q^2) = 2\sum_{q\in\{u,d,\dots\}}\left(xq(x,Q^2) + x\bar{q}(x,Q^2)\right)$$

$$xF_3(x,Q^2) = 2\sum_{q\in\{u,d,\dots\}}\left(xq(x,Q^2) - x\bar{q}(x,Q^2)\right)$$

$F_1 = \frac{F_2}{2x}$ via the Callan-Gross relation

q and $\bar{q}$ are the (anti) quark PDFs

The important things to remember about neutrino DIS:

1. F_3 is <u>unique</u> to neutrino DIS.
   It represents interference between the vector and axial parts of the weak interaction.
   F_1 and F_2 are measurable in electron DIS

2. F_3 is sensitive to the <u>valence</u> quark content of the nucleon, which leads to the sign difference for ν v.s. $\bar{\nu}$.

   As neutrino energy increases, scattering off of <u>sea</u> quarks dominates. So:

   $\sigma_{\bar{\nu}} < \sigma_{\nu}$ at lower E_ν

   $\sigma_{\bar{\nu}} \approx \sigma_{\nu}$ at higher E_ν

3. For NC scattering, the F_i structure functions depend on the neutral current vector and axial couplings of the quarks.
   In general, $\sigma_{NC} \lesssim \sigma_{CC}/2$

④ At ultra-high energies, the W/Z propagators become on-shell!

$$\frac{1}{\left(1+\frac{Q^2}{M_{W/Z}^2}\right)^2} \approx \begin{cases} \left(\frac{M_{W/Z}}{Q}\right)^4 & Q^2 \gg M_{W/Z}^2 \\ 1 & Q^2 \ll M_{W/Z}^2 \end{cases}$$

transition happens at $E_\nu \sim 100$ TeV

upshot: $\sigma(E_\nu) \propto \begin{cases} E_\nu & E_\nu \lesssim 100\,\text{TeV} \\ E_\nu^{0.363} & E_\nu \gtrsim 100\,\text{TeV} \end{cases}$

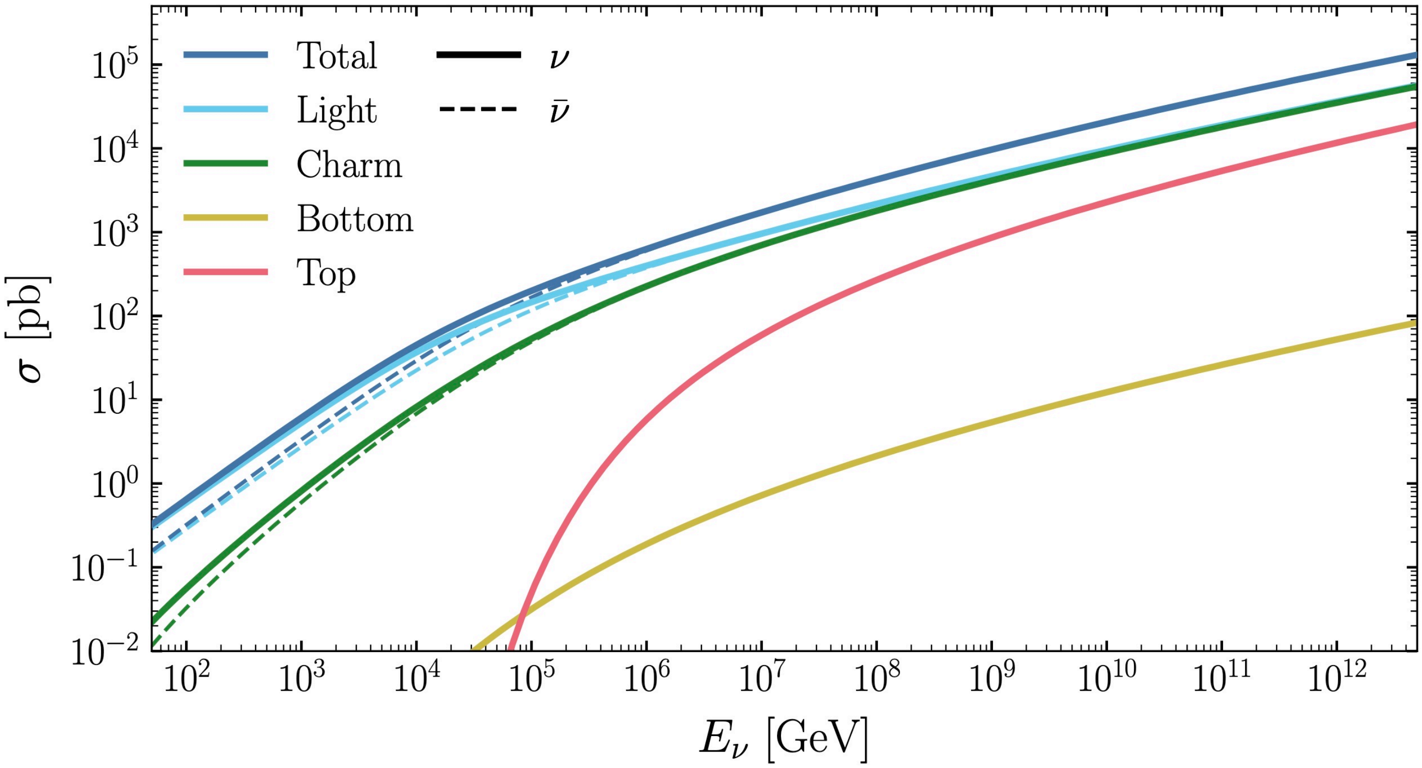
$x = Q^2/M^2$

$$(1+x)^{-2} \approx 1 - \left[2(1+x)^{-3}\right]_{x=0} x \approx 1 - \frac{2Q^2}{M^2}$$

$x = M^2/Q^2$

$$\frac{x^2}{(x+1)^2} \approx \frac{1}{2}\left[\frac{2}{(x+1)^2}\right]_{x=0} x^2 \approx \frac{M^4}{Q^4}$$

Points (2) and (4) are evident here:



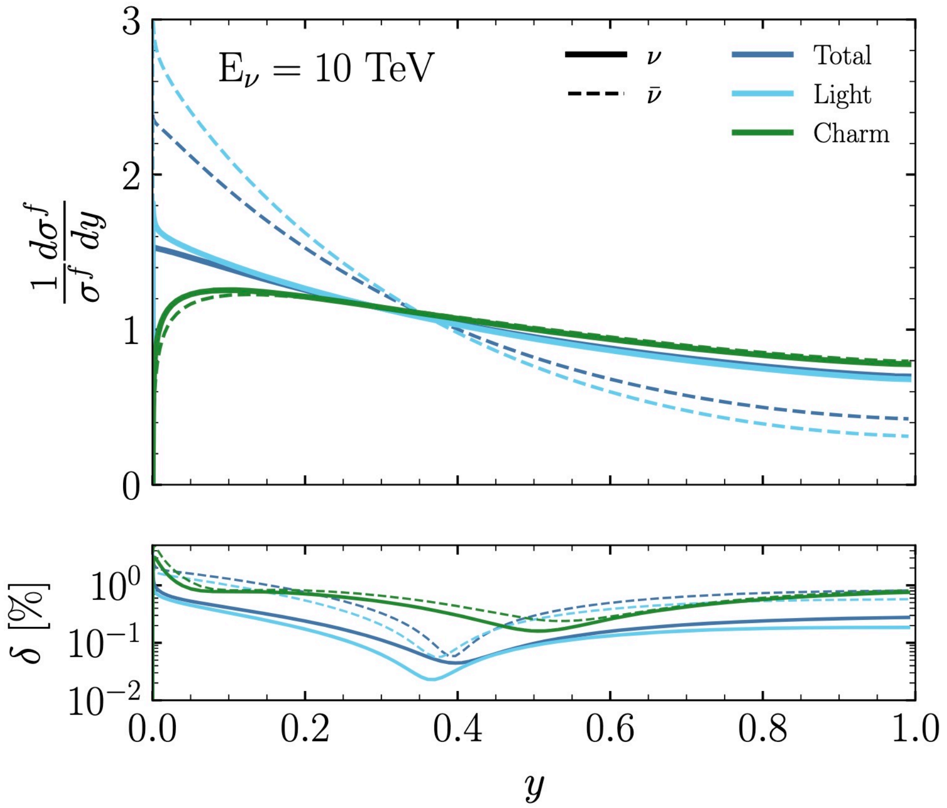
Weigel, Conrad, Garcia-Soto 2024

(5) Inelasticity is the most important variable from a labratory perspective

$$y = \frac{E_{had}}{E_\ell + E_{had}}$$

all measurable in IceCube, for example

importantly, the $\pm$ difference for F_3 means that ν and $\bar{\nu}$ have different inelasticity distributions



Weigel, Conrad, Garcia-Soto 2024

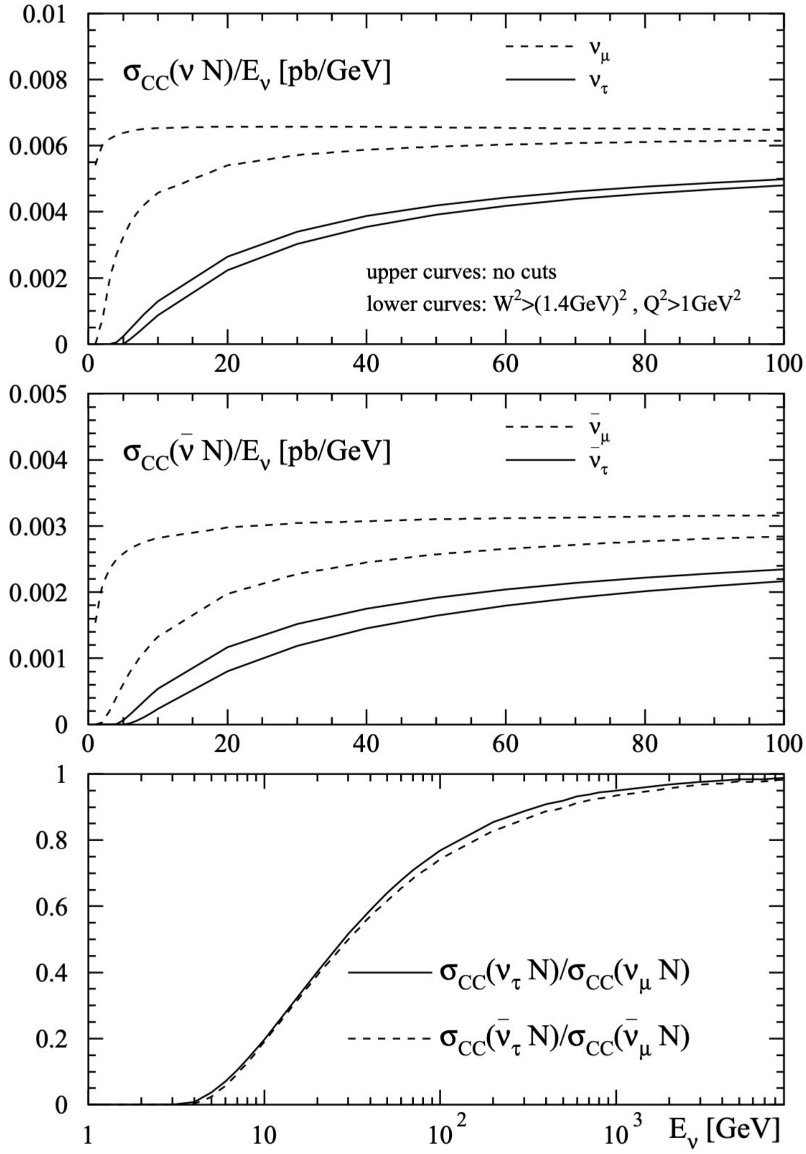
(6) Different neutrino flavors have different CC cross sections due to the charged lepton mass:

$$\sigma^{CC}_{\nu_e} > \sigma^{CC}_{\nu_\mu} > \sigma^{CC}_{\nu_\tau}$$

$$\xrightarrow[E_\nu \to \infty]{} \sigma^{CC}_{\nu_e} \approx \sigma^{CC}_{\nu_\mu} \approx \sigma^{CC}_{\nu_\tau}$$

Even though the threshold is at $E_\nu = E_\ell$, suppression lasts for a while

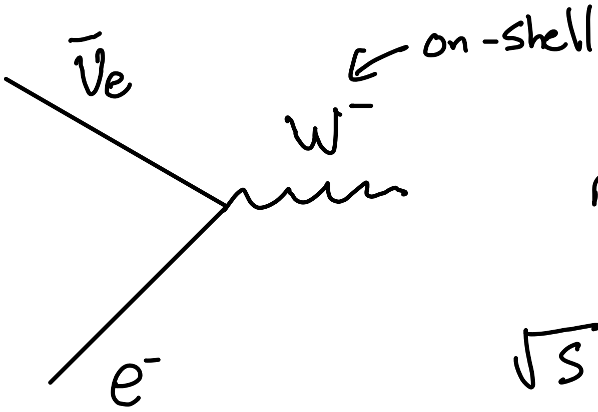
Kretzer, Reno 2002



7 The Earth can shield neutrinos at sufficiently high energy!

$$\underbrace{P_{interact}(E_\nu)}_{\text{Probability that a neutrino interacts while traversing the Earth}} = \exp\Big[-\underbrace{\underbrace{\sigma_{DIS}(E_\nu)}_{\nu\text{ DIS cross section [cm}^2]}\underbrace{\langle n^{\oplus}_{targets}\rangle}_{\text{average Earth target density }[\frac{\text{nucleons}}{\text{cm}^3}]}\underbrace{D_\oplus}_{\text{Earth diameter [cm]}}}_{\sim 1 \text{ @ } E_\nu \sim 10\text{-}100 \text{ TeV}}\Big]$$

One last interaction: the Glashow resonance!

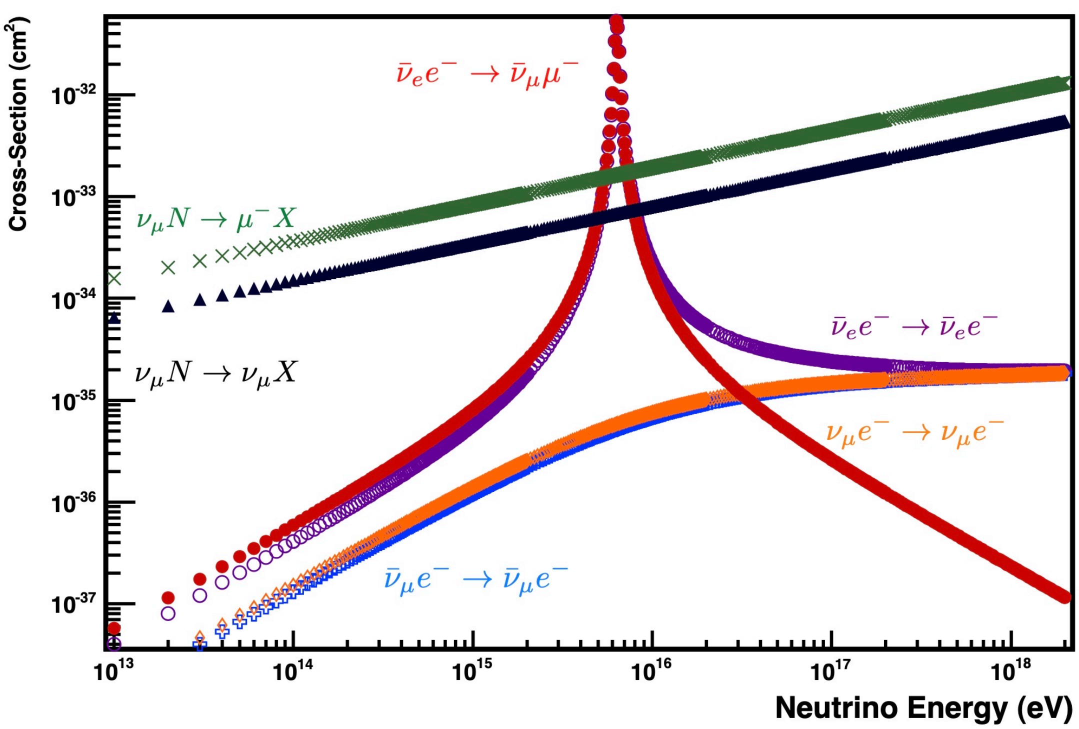


resonance condition:

$$\sqrt{s} = \sqrt{2E_\nu m_e} \approx M_W$$

$$\Rightarrow E_\nu \approx 6.3 \text{ PeV}$$

To summarize the high energy landscape:



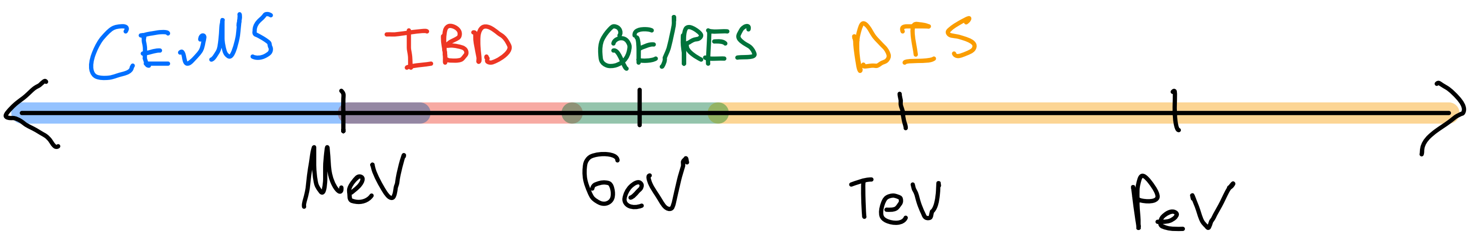
Formaggio, Zeller 2013

Next time.

What does a ν DIS interaction look like in IceCube?

$\nu_e N \to e X$	$\nu_\mu N \to \mu X$	$\nu_\tau N \to \tau X$	$\nu N \to \nu X$
"cascade"	"track"	"(double) cascade"	NC cascade

All interactions are relevant for IceCube!



Supernova Neutrinos

LE Atmospherics (oscillations)

Astro neutrinos

HE Atmospherics