Effective Area / Acceptance Calculation & Topography

TAMBO-TRINITY-BEACON Workshop

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10/30/25

Point Source Effective Area

- Number of neutrinos observed N_{obs} given a flux of tau neutrinos ϕ_{ν_τ}:

$$\frac{dN_{\mathrm{obs}}}{dt\, d\mathcal{E}_\nu\, d\Omega\, dA} = (\hat{r} \cdot \hat{u})\ \Theta(\hat{r} \cdot \hat{u})\ \Phi_{\nu_\tau}(t, \mathcal{E}_\nu, \hat{r})\ P_{\mathrm{obs}}(t, \mathcal{E}_\nu, \hat{r}, \hat{u}))$$

dt = differential time interval

$d\mathcal{E}_\nu$ = differential neutrino energy

$d\Omega$ = differential solid angle

dA = differential area

$\hat{r}$ = vector from neutrino source to Earth

$\hat{\mathrm{u}}$ = vector from center of Earth to differential area

Θ = Heaviside step function

P_{obs} = probability of detection

* We define $\hat{r}$ as extending from the source so that $\hat{r} \cdot \hat{u}$ is positive for below-horizon sources

Point Source Effective Area

- A ν_τ flux originating from a point source is modeled as:

$$S(t, \mathcal{E}_\nu, \hat{r}_*) = \int d\Omega\ \delta(\hat{r} - \hat{r}_*)\ \Phi_{\nu_\tau}(t, \mathcal{E}_\nu, \hat{r})$$

Integrate over solid angle →

$$\frac{dN_{\text{obs}}}{dt\ d\mathcal{E}_\nu\ dA} = (\hat{r}_* \cdot \hat{u})\ \Theta(\hat{r}_* \cdot \hat{u})\ S(t, \mathcal{E}_\nu, \hat{r}_*)\ P_{\text{obs}}(t, \mathcal{E}_\nu, \hat{r}_*, \hat{u}))$$

S does not depend on *dA* →

$$\frac{dN_{\text{obs}}}{dt\ d\mathcal{E}_\nu} = S(t, \mathcal{E}_\nu, \hat{r}_*)\ A(t, \mathcal{E}_\nu, \hat{r}_*)$$

Point Source Effective Area: $A(t, \mathcal{E}_\nu, \hat{r}_*) = \int_{A_g} dA_g\ (\hat{r}_* \cdot \hat{u})\ \Theta(\hat{r}_* \cdot \hat{u})\ P_{\text{obs}}(t, \mathcal{E}_\nu, \hat{r}_*, \hat{u})$

Acceptance

- For a diffuse, isotropic flux the flux does not depend on direction or time:

$$\frac{dN_{\text{obs}}}{dt\, d\mathcal{E}_\nu} = \Phi_{\nu_\tau}(\mathcal{E}_\nu)\, \langle A\Omega \rangle(t, \mathcal{E}_\nu)$$

Acceptance: $$\langle A\Omega \rangle(t, \mathcal{E}_\nu) = \int_\Omega d\Omega \int_{A_g} dA_g\; (\hat{r}\cdot\hat{u})\, \Theta(\hat{r}\cdot\hat{u})\, P_{\text{obs}}(t, \mathcal{E}_\nu, \hat{r}, \hat{u})$$

- Acceptance is equivalent to integrating the point source effective over solid angle

Neutrino Monte Carlo

- **M**ultiple **A**ntenna A**r**rays on **Mo**untains **T**au **S**ensitivity
- Monte Carlo which calculates the effective area of any configuration of mountaintop phased radio arrays to point sources of neutrinos
- Accounts for the effective areas of individual stations overlapping

$$A(t, E_\nu, \hat{r}) \approx \frac{A_g}{N} \sum_{i=1}^{N} (\hat{r}_i \cdot \hat{u}_i)\, \Theta(\hat{r}_i \cdot \hat{u}_i)\, P_{i,\mathrm{exit}}\, P_{i,\mathrm{detect}}$$

Geometric Area in View (A_g)

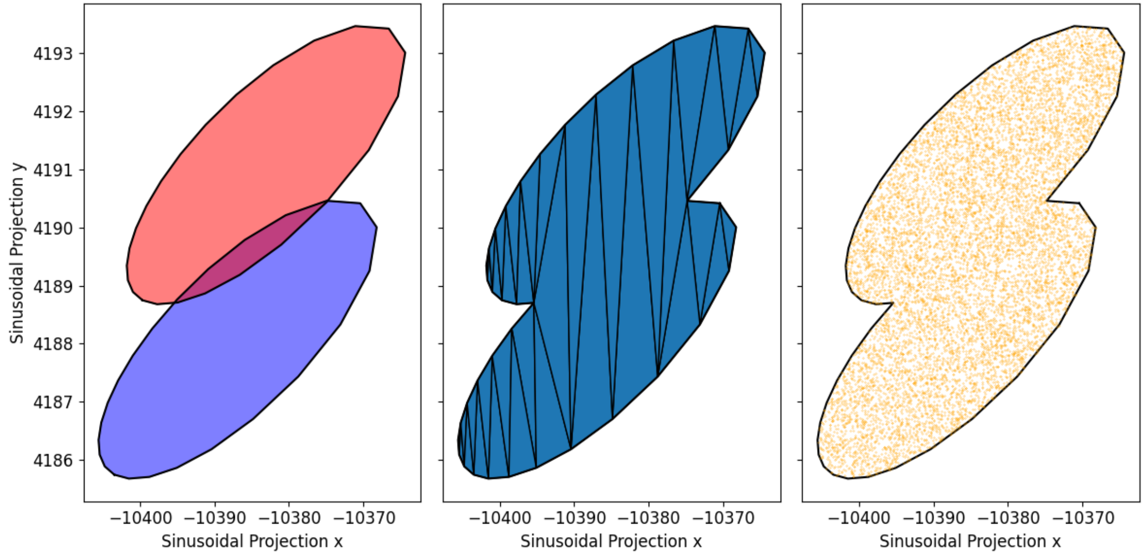
\# of Events Generated (N)

Probability that τ exits the Earth ($P_{i,\mathrm{exit}}$)

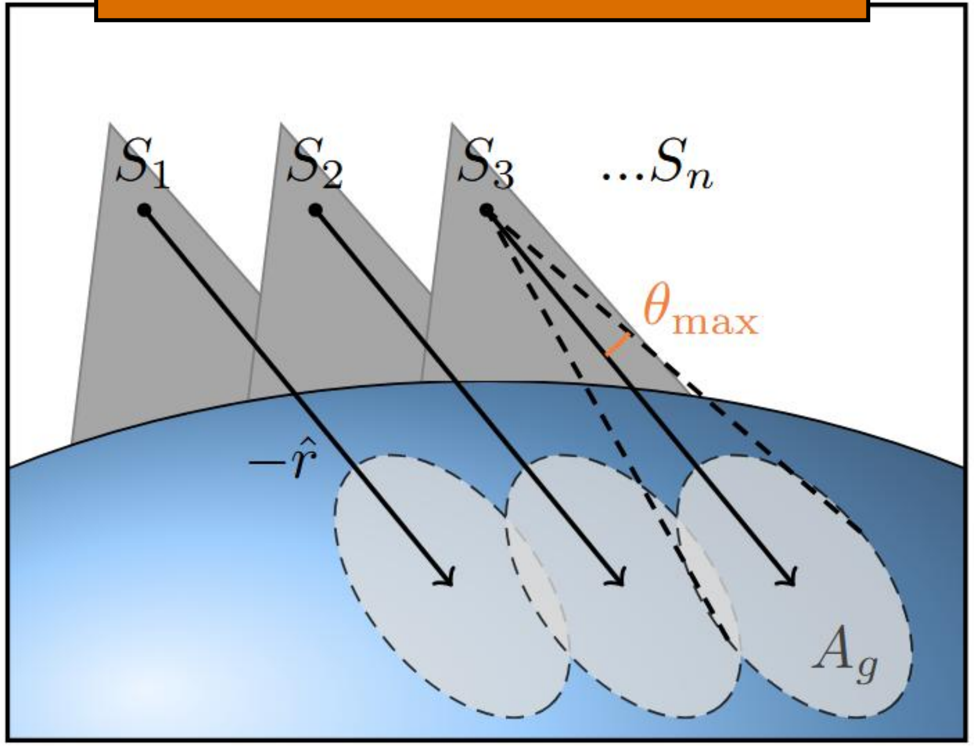
Probability that the resulting shower is detected ($P_{i,\mathrm{detect}}$)

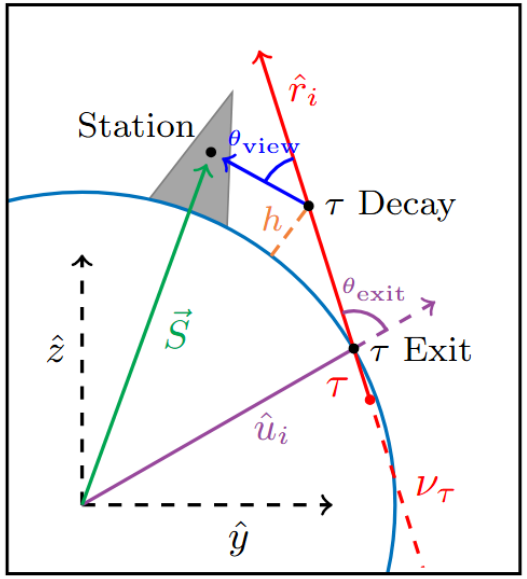
Geometry

Union of all the areas = A_g. Uniformly sample tau exit points ($\hat{u}_i$)



From each station, find where a cone in the source direction $-\hat{r}$ intersects Earth



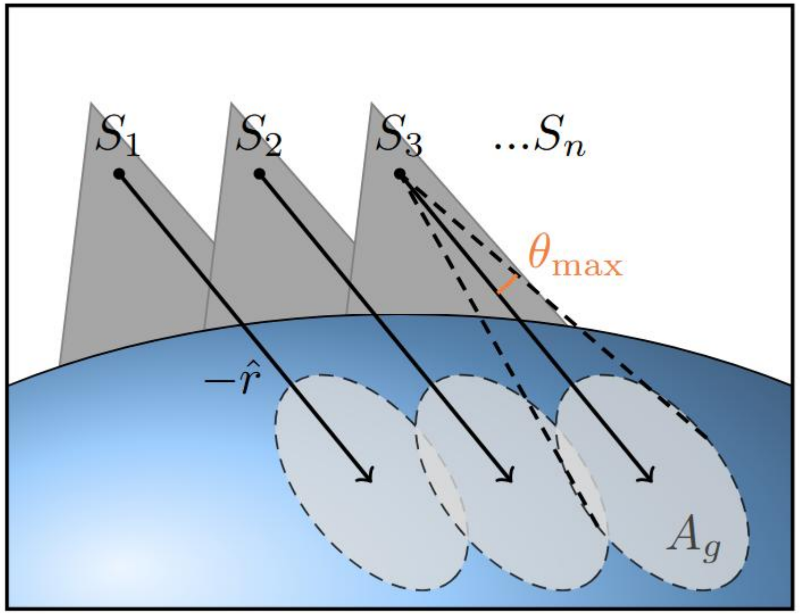


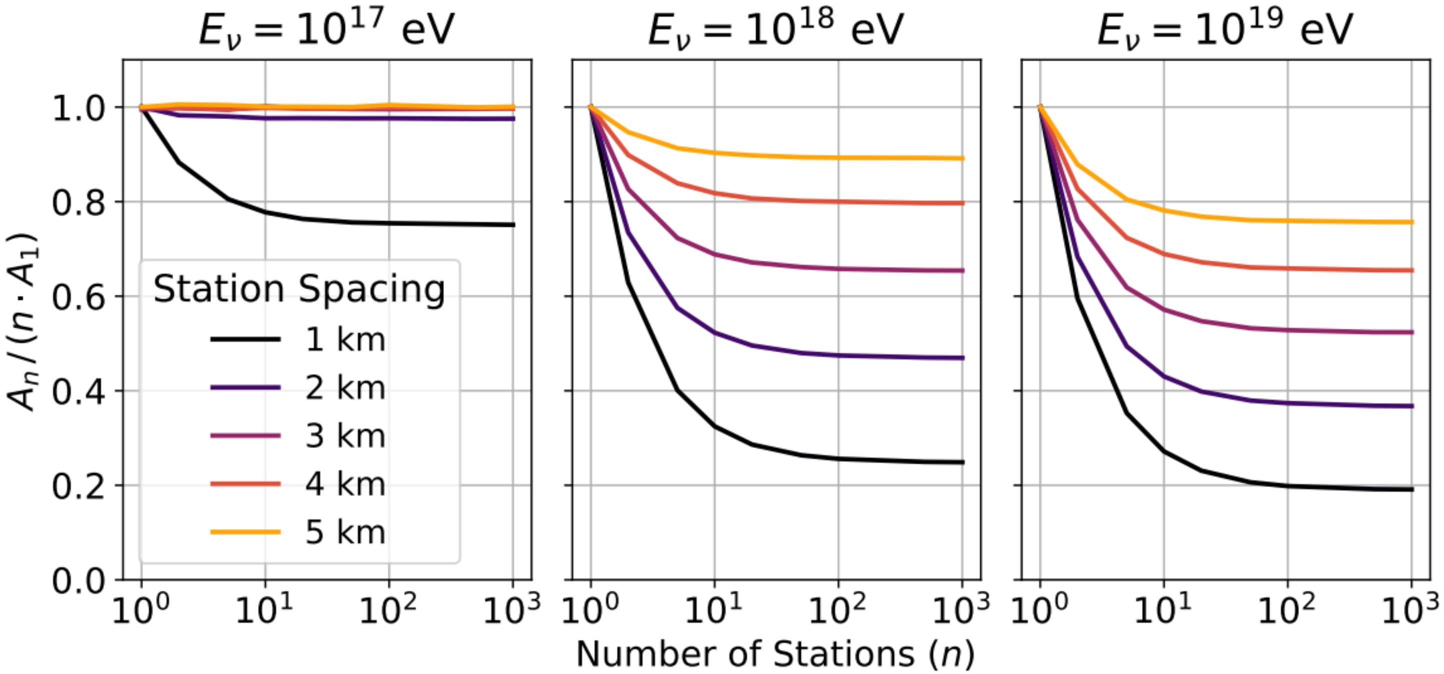
We now have:

- A_g
- $\widehat{r_i}$
- $\widehat{u_i}$

The Effect of Overlap

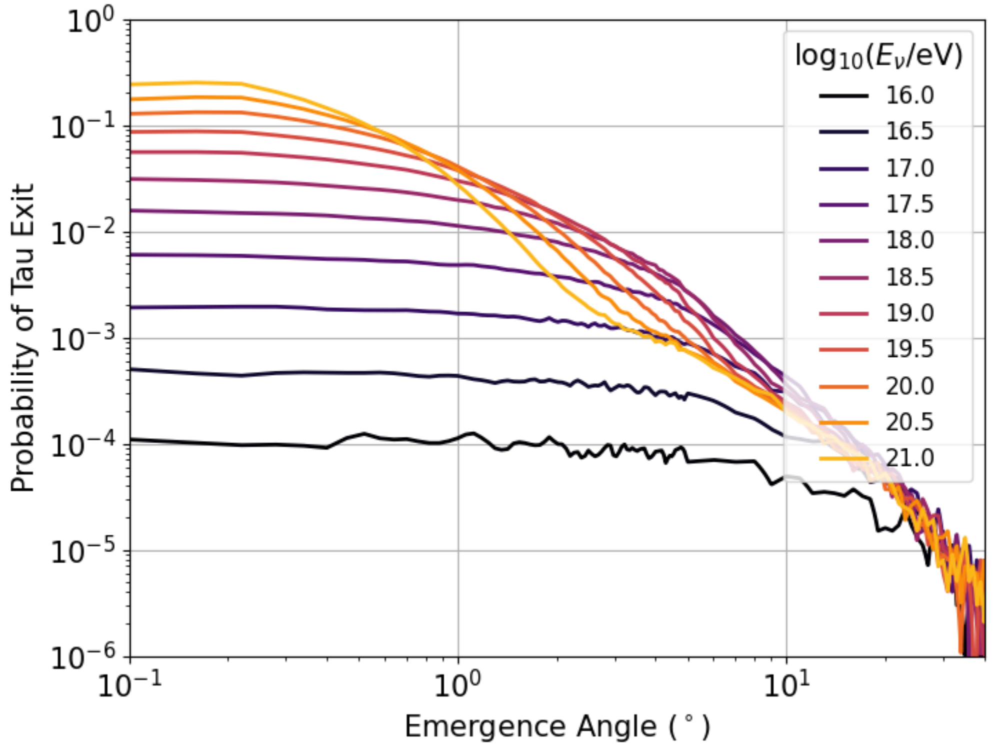
- If the effective areas of individual stations do not overlap, then the overall effective area scales linearly with the number of stations
- If overlap is present, then $A_n < n \cdot A_1$

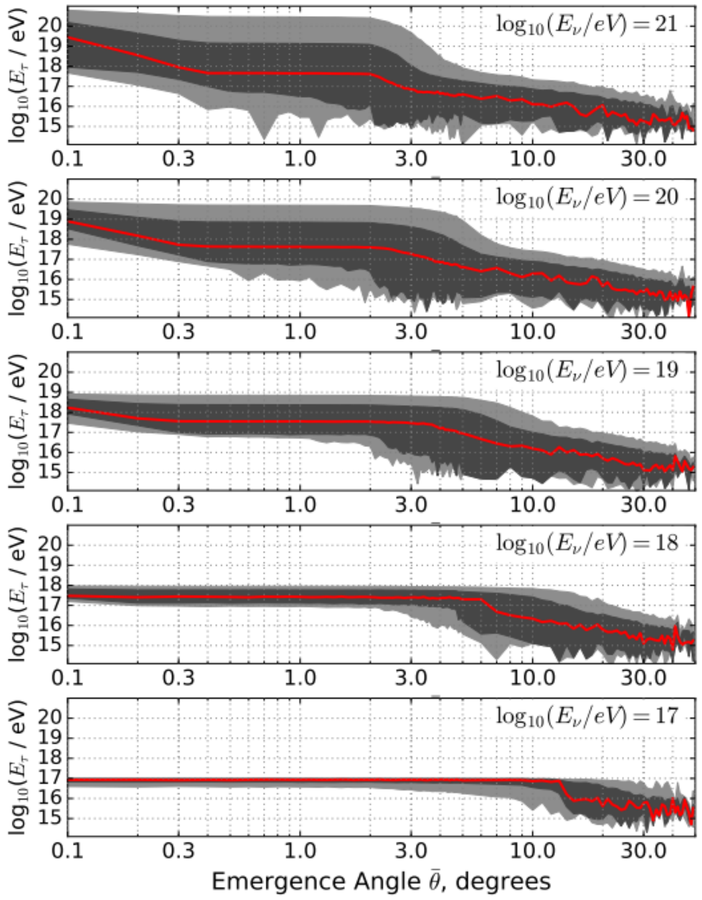




Exit Probability and τ Energy

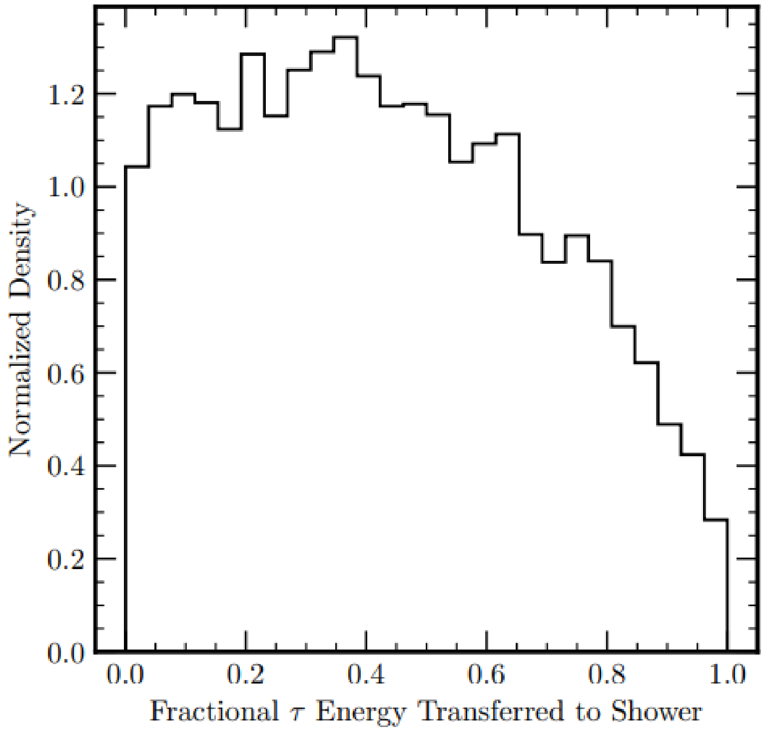
- P_{exit} and the resulting τ-lepton energies are sampled using look-up tables generated using NuTauSim





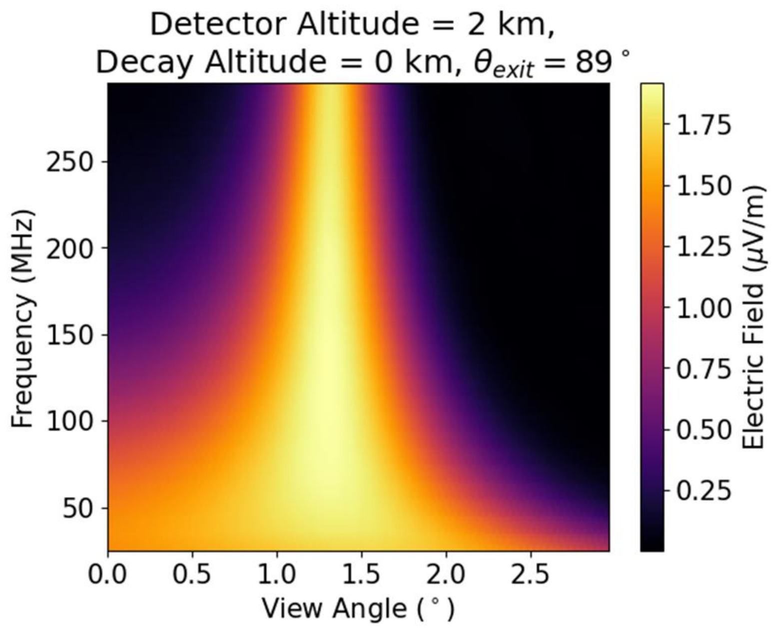
τ-Lepton Decay

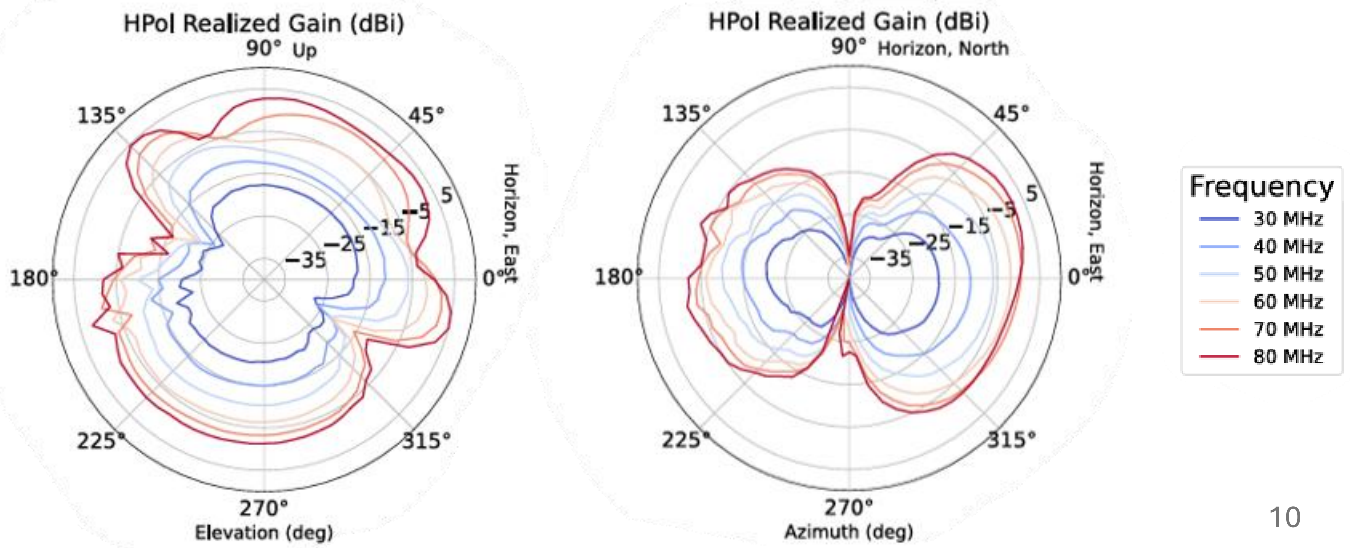
- Decay distances are sampled from an exponential distribution given the τ lifetime and energy
- Extensive air shower energies are sampled from distributions generated with PYTHIA
- Decays which occur behind a station are considered undetectable



Detection Probability

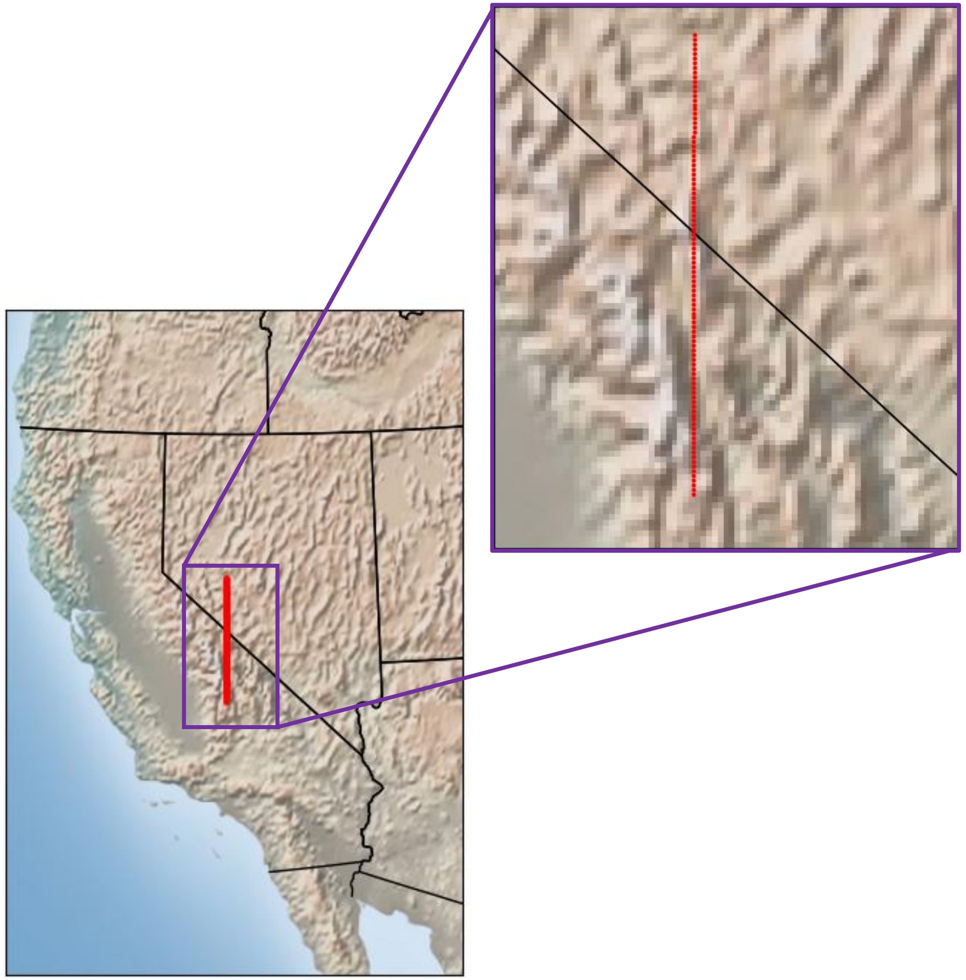
- Peak electric field amplitude of each EAS is interpolated from look-up tables generated using ZHAireS-RASPASS
- Electric fields are converted to voltages given an antenna model
- If the signal-to-noise ratio exceeds an input threshold, for any of the stations, then an event has been detected ➔ $N_{\text{detected}}/N = P_{\text{detect}}$





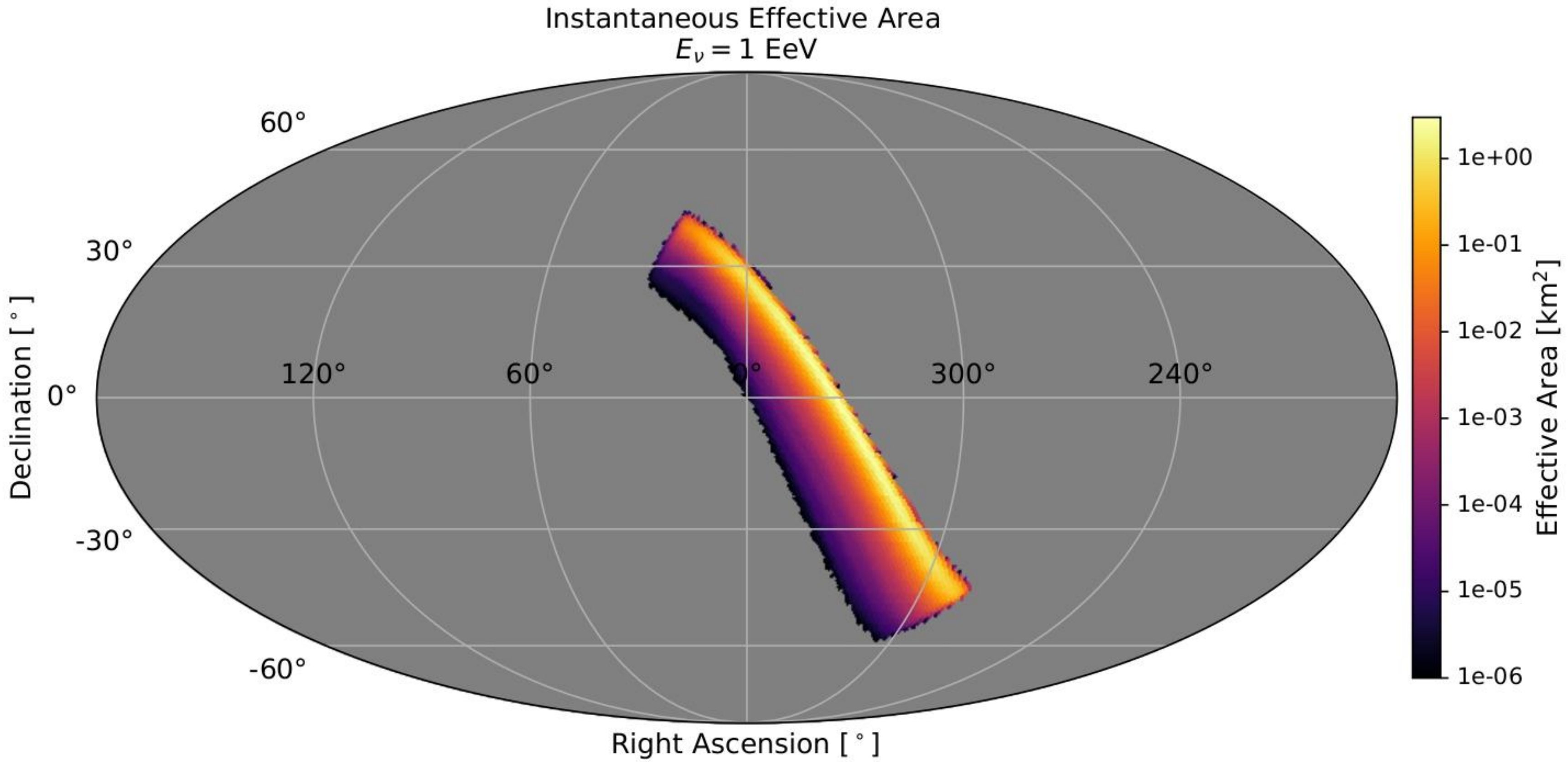
An Example

- 100 stations consisting of 10 phased antennas
- Spaced 3 km apart along same longitude, centered on location of BEACON Prototype
- 3 km altitude
- Facing East, 120° FoV
- SNR = 5 trigger



Instantaneous Effective Area

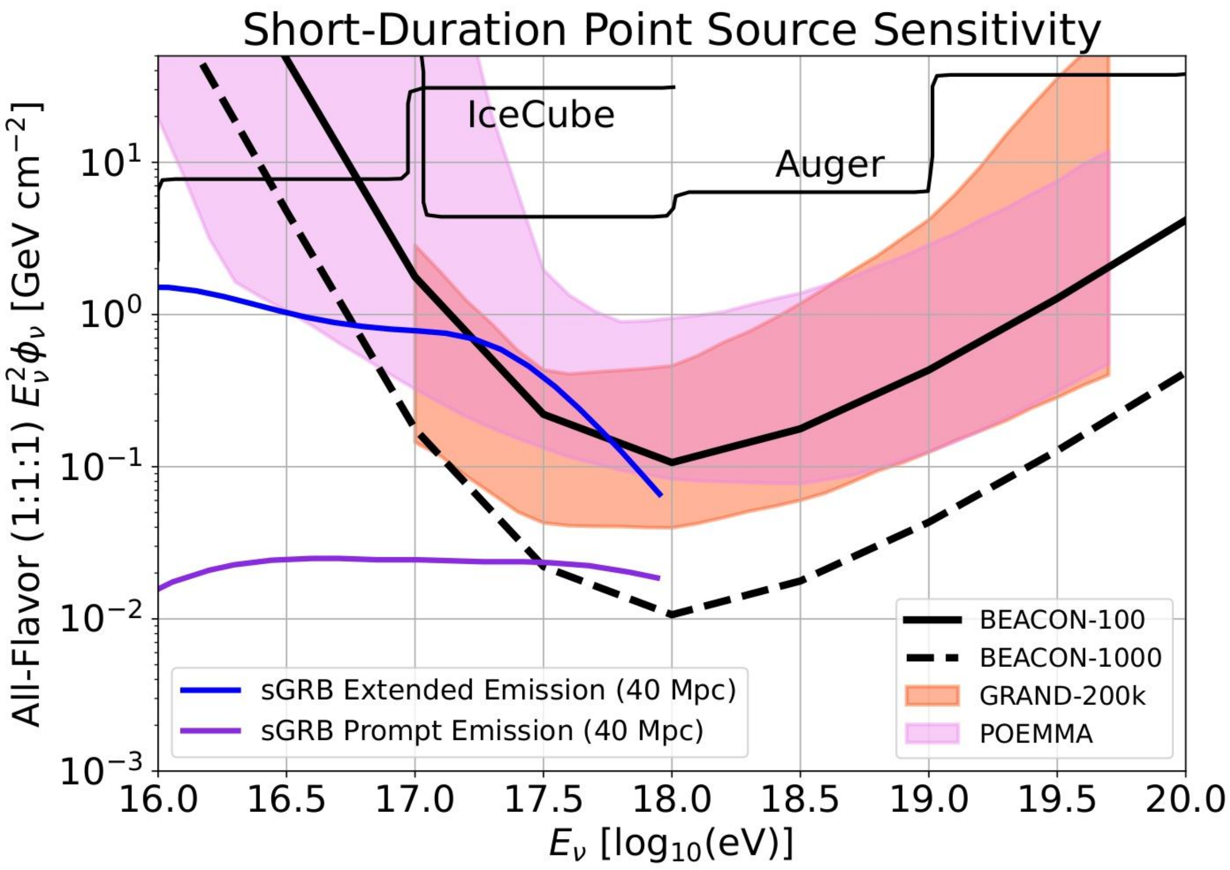
- BEACON has a large instantaneous effective area along the horizon



Short-Duration Point Source Sensitivity

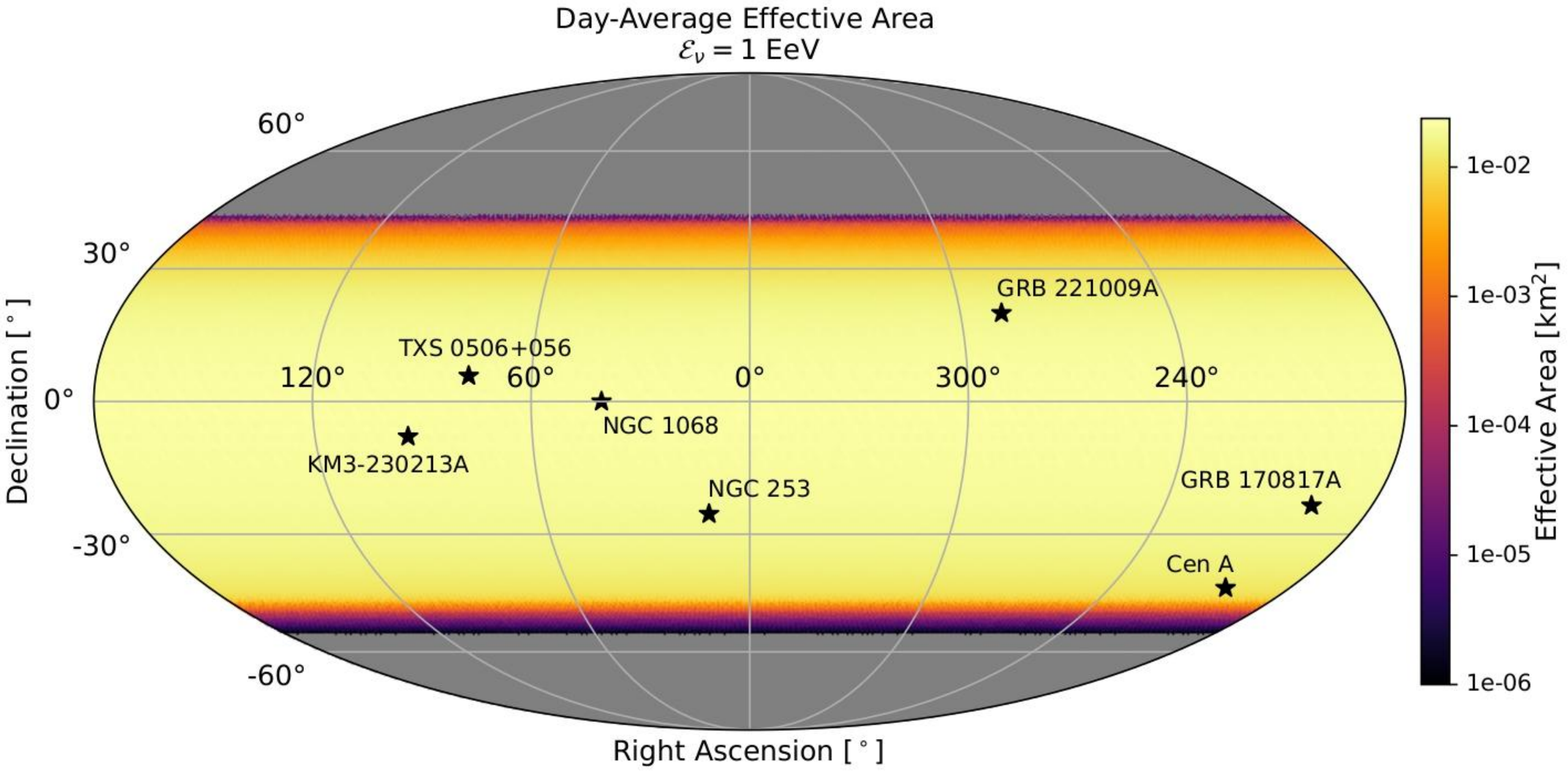
Sensitivity to an all-flavor $dN_\nu/d\mathcal{E}_\nu \propto \mathcal{E}_\nu^{-2}$ neutrino fluence ϕ_ν:

$$\mathcal{E}_\nu^2 \phi_\nu = \frac{2.44}{\ln(10)} \frac{3\,\mathcal{E}_\nu}{A(\mathcal{E}_\nu)}$$



Day-Average Effective Area

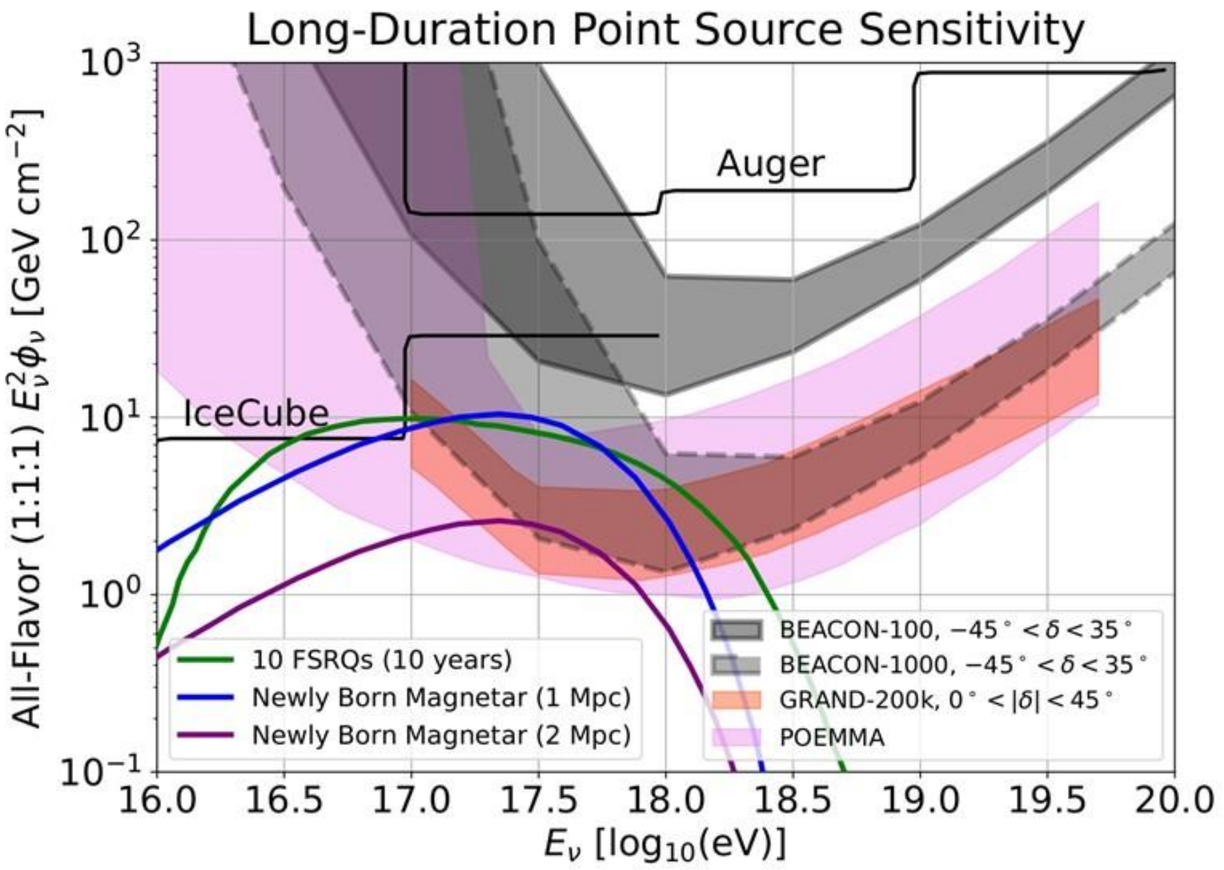
- BEACON observes ~70% of the sky each day



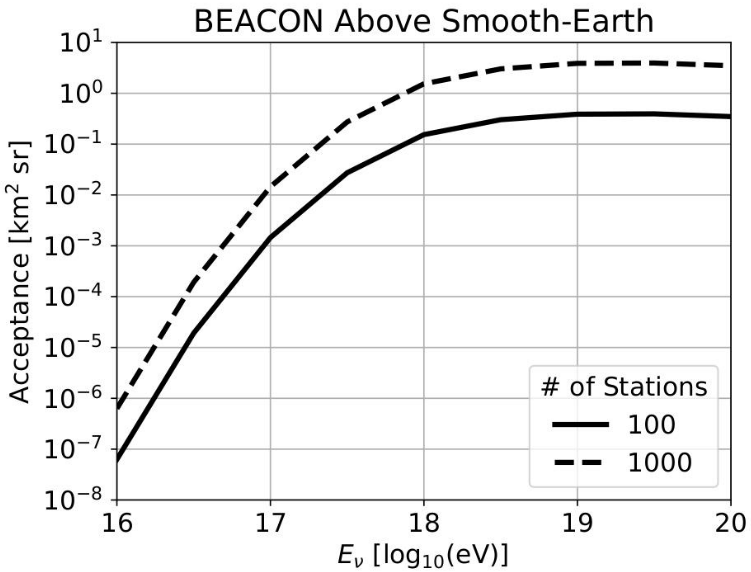
Long-Duration Point Source Sensitivity

Sensitivity to an all-flavor $dN_\nu/d\mathcal{E}_\nu \propto \mathcal{E}_\nu^{-2}$ neutrino fluence ϕ_ν:

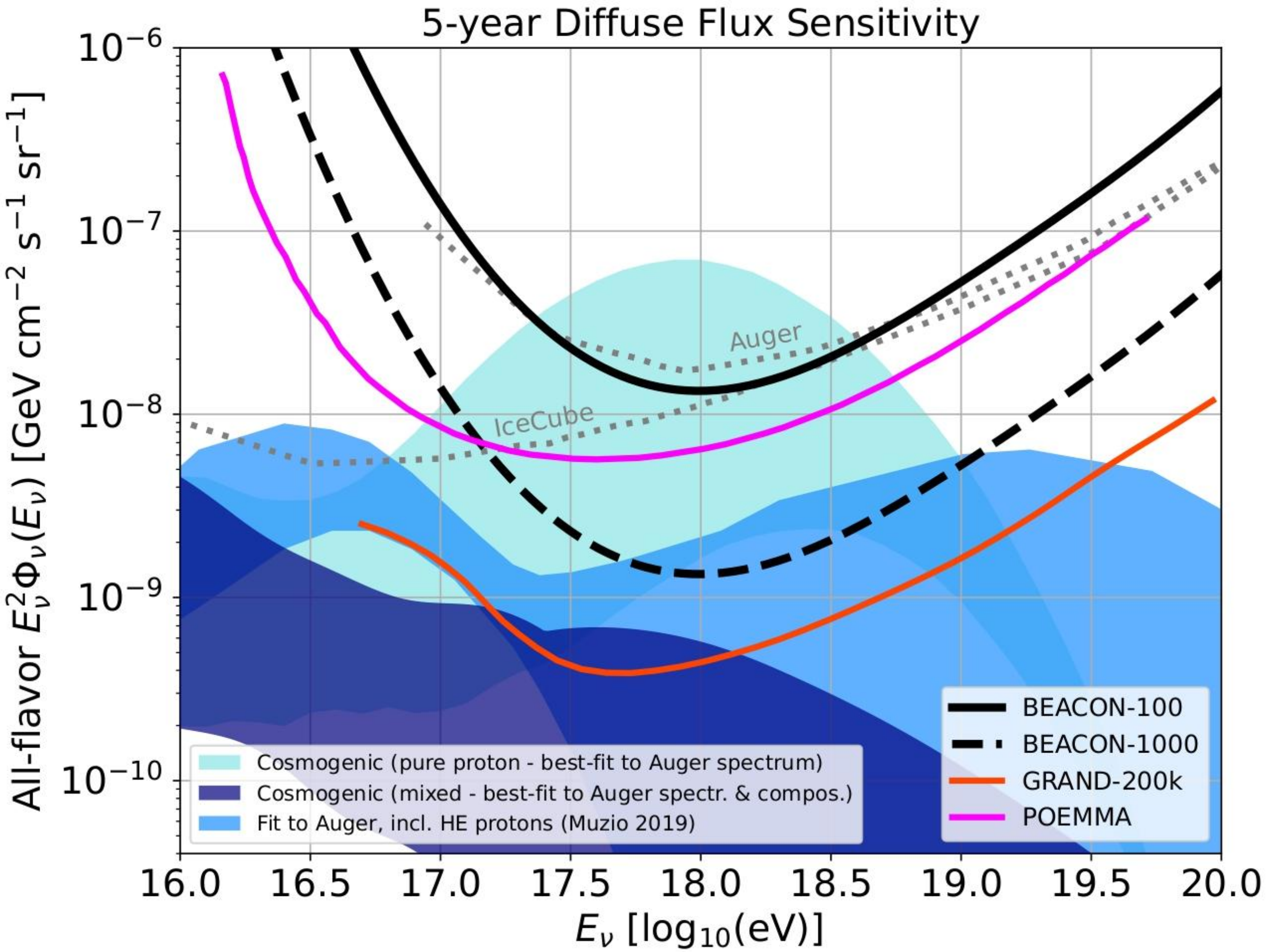
$$\mathcal{E}_\nu^2 \phi_\nu = \frac{2.44}{\ln(10)} \frac{3\,\mathcal{E}_\nu}{A(\mathcal{E}_\nu)}$$



Diffuse Flux Sensitivity



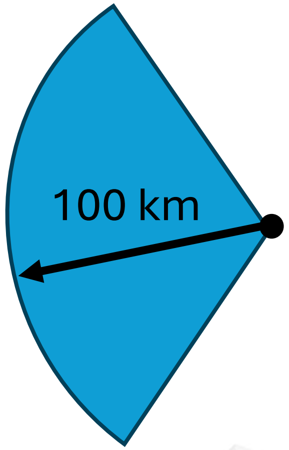
$$\mathcal{E}_\nu^2 \Phi_\nu = \frac{2.44}{\ln(10)} \frac{3\,\mathcal{E}_\nu}{\langle A\Omega \rangle (\mathcal{E}_\nu)\, T}$$

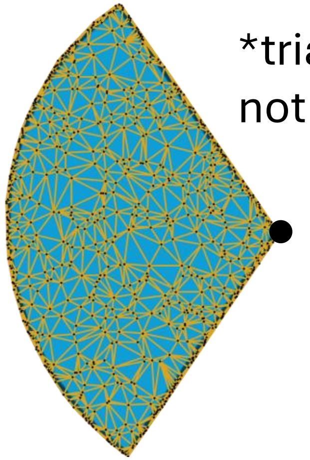


Adding Topography to MARMOTS

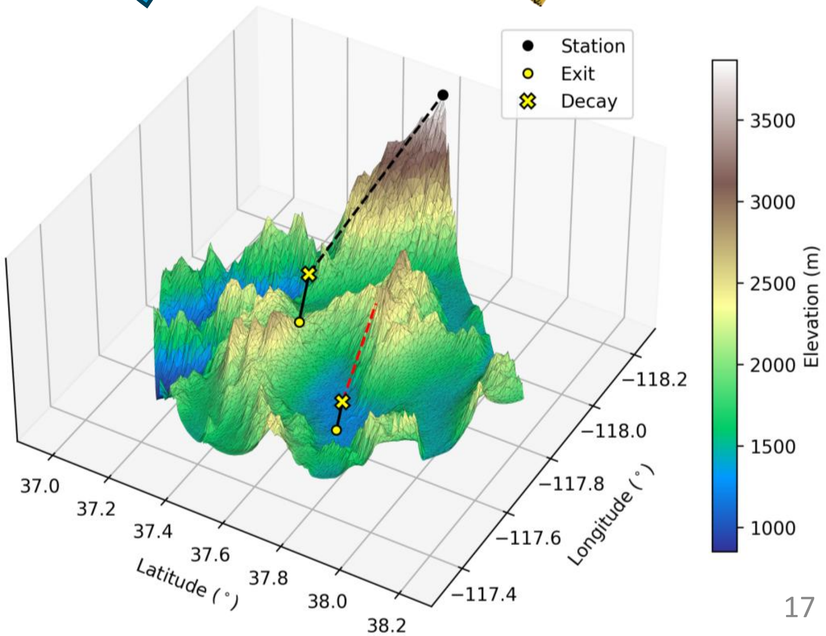
- Topography can help or hurt the effective area
  - Mountains and canyon walls provide additional surface area for exiting tau-leptons
  - Nearby features can block radio line-of-sight
- To model topography:
  1. Triangulate the field-of-view of each station
  2. At each triangle vertex, interpolate elevation from the SRTM database given the latitude/longitude

  ➔ Triangulated mesh of the Earth's surface



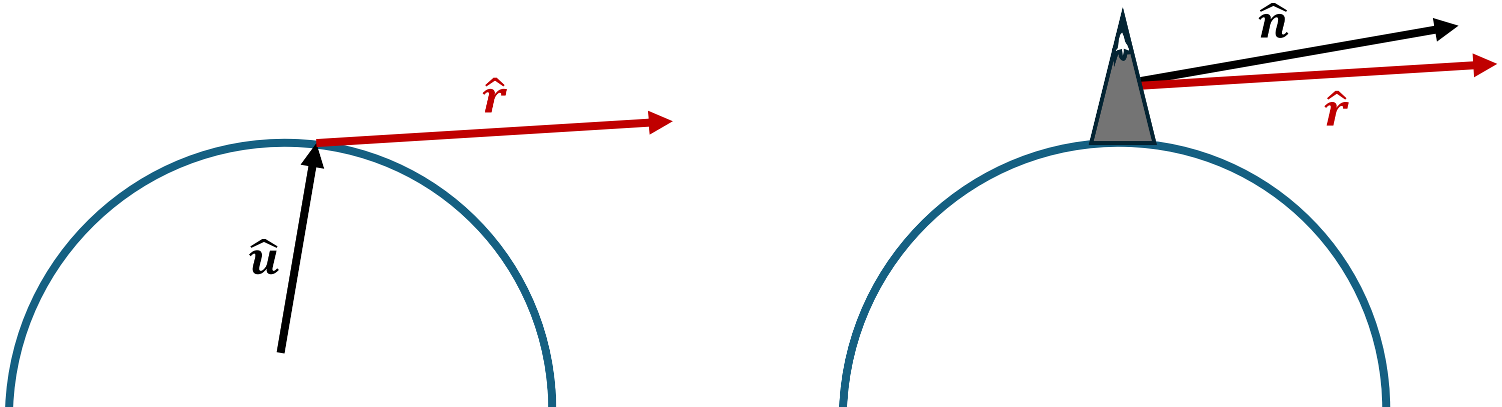


*triangles not to scale



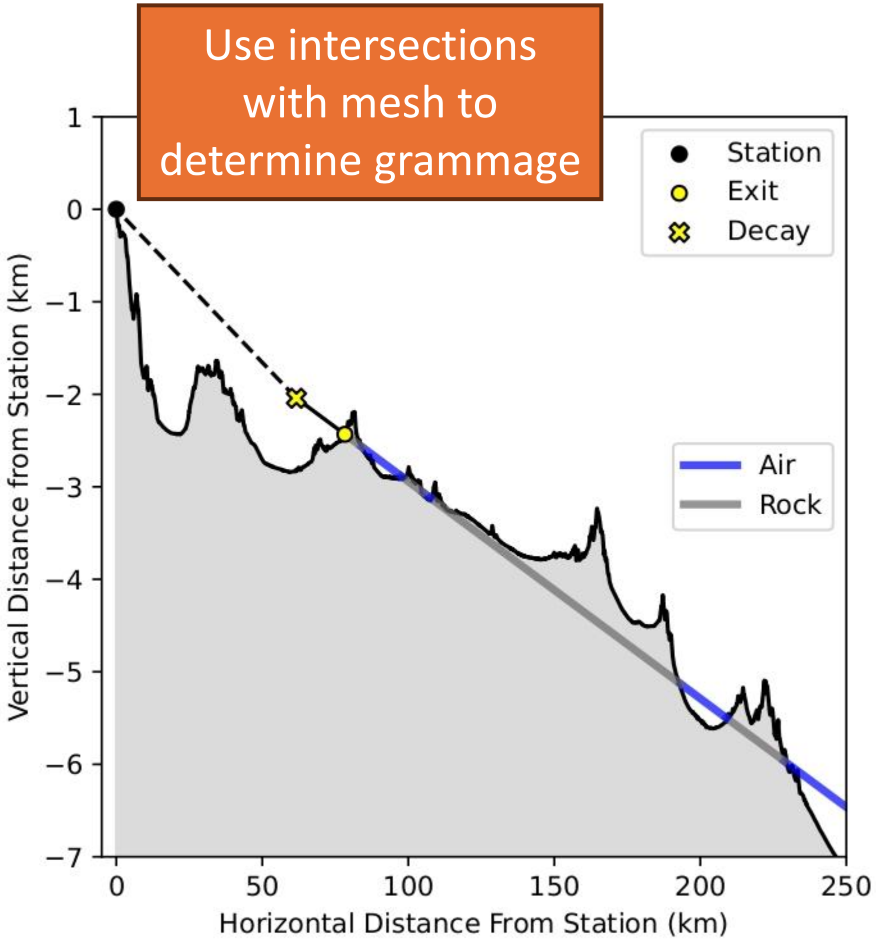
Geometry: Topography

- Just like before, a cone is directed from each station in the direction of the point source
- This time, we look for triangles that exist within the cone
- A_g is the sum of the areas of these triangles
- Exit points are then uniformly sampled across these triangles
- $(\hat{r}_i \cdot \hat{u}_i)$ is replaced with $(\hat{r}_i \cdot \hat{n}_i)$, where $\hat{n}_i$ is the normal vector of the triangle on which the tau exit point was sampled



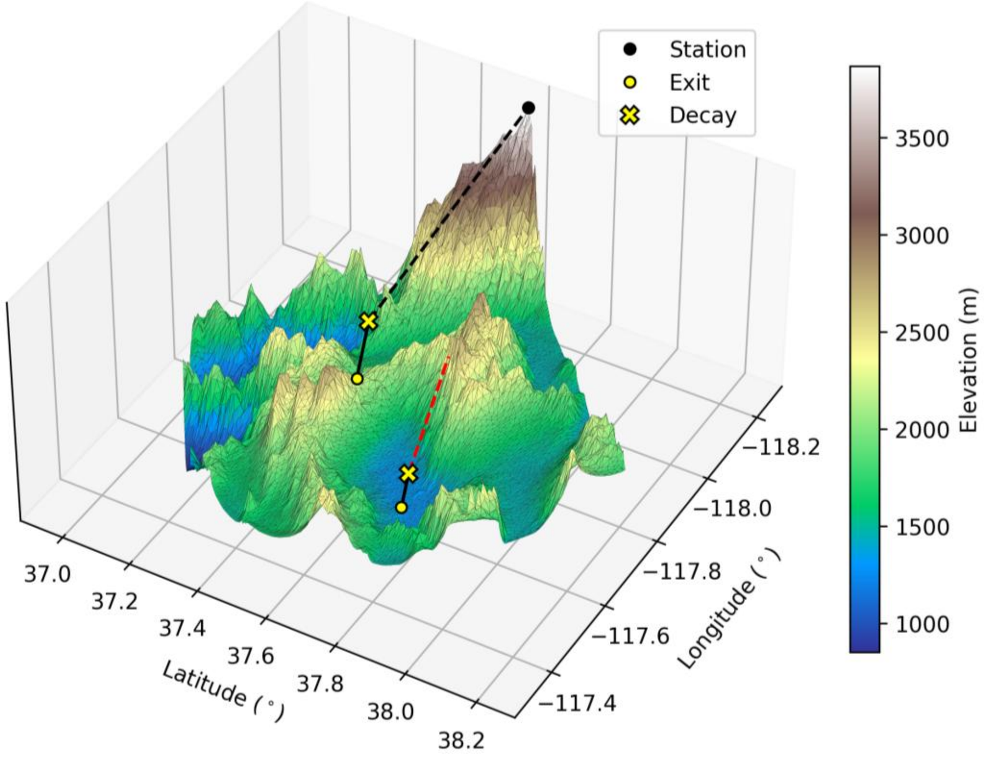
Exit Probability: Topography

- From each exit point, we find where the particle axis intersects the surface mesh in the backwards direction
- Intersections are found using a Bounding Volume Hierarchy and the Möller–Trumbore intersection algorithm
- Intersections determine when the τ is traversing air versus rock
- From the total grammage traversed, we can determine P_{exit}

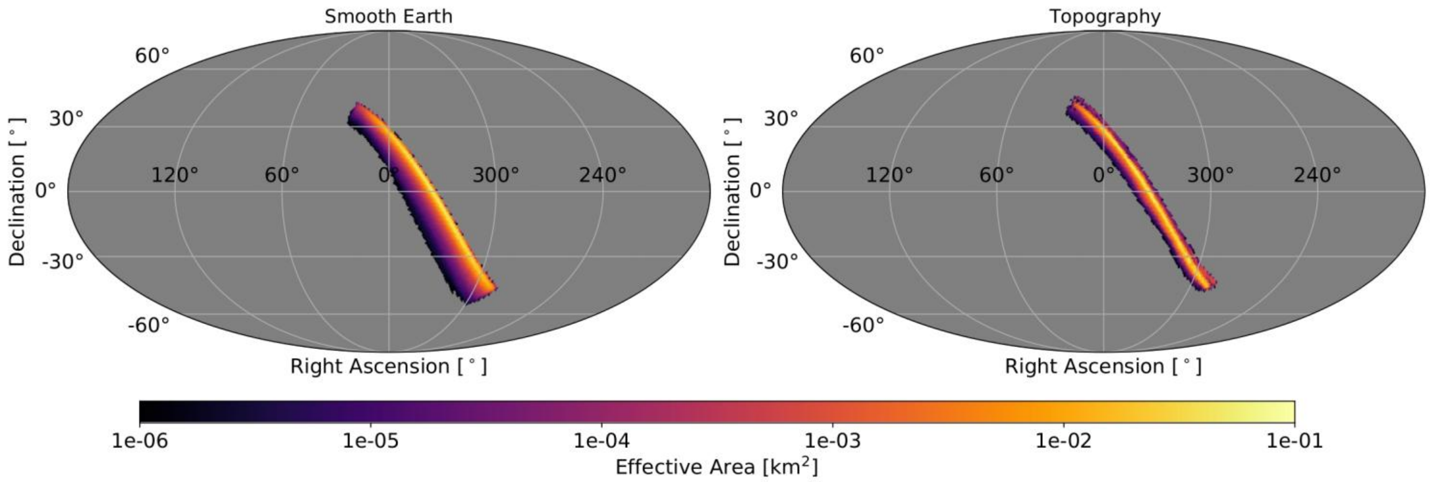


Detection Probability: Topography

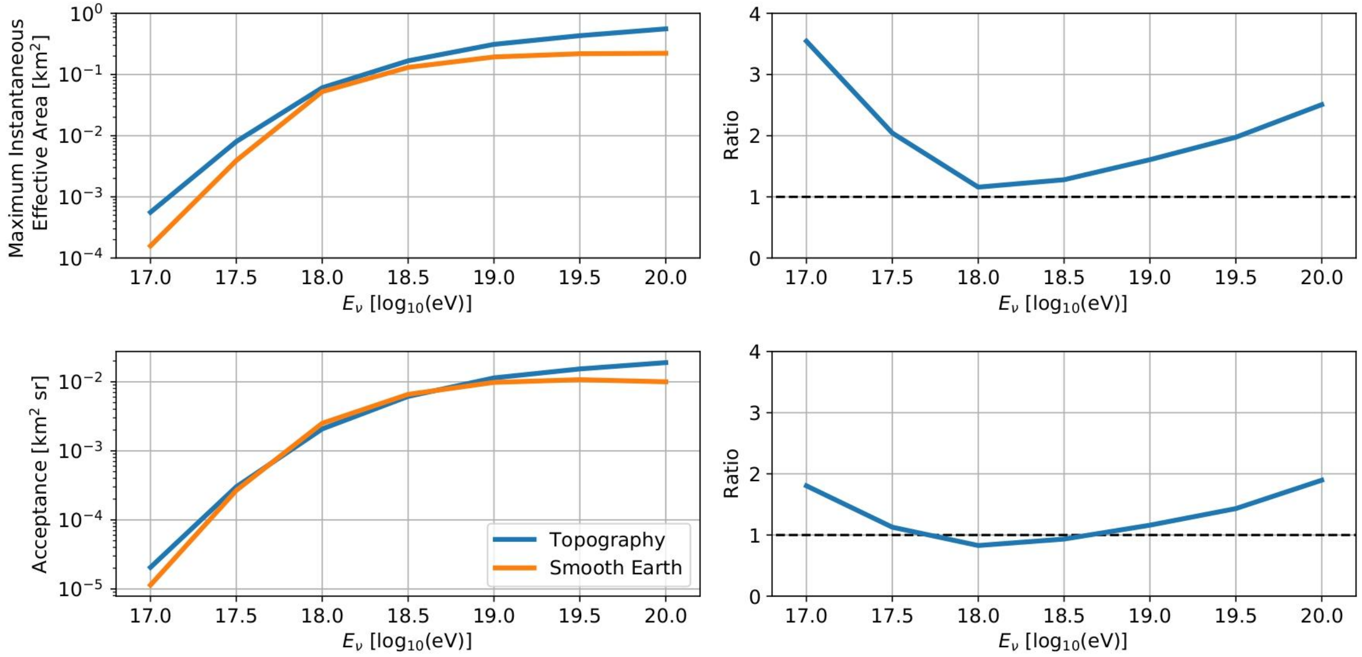
- From each decay point, we find if the particle axis intersects the surface mesh in the forward direction
- If an intersection exists, then radio line-of-sight is interrupted → event is not detected
- If no intersection is present, then we check for a trigger



Effect of Topography



- Simulation of single station at BEACON prototype site with and without real topography
- Topography increases the max effective area by factor of 1-3
- Effect is site and energy dependent



Adapting MARMOTS for Other Experiments

- Radio is only relevant during the calculation of P_{detect}, the last step of the simulation → only small modifications needed for prior modules

- To calculate P_{detect} for non-radio experiments using MARMOTS, we must replace the radio emission models with models of Cherenkov emission, scintillation, etc.

- If you're interested in adapting MARMOTS: https://github.com/beaconTau/marmots

Summary

- MARMOTS is a Monte Carlo originally developed to calculate the point source effective area of an arbitrary configuration of elevated phased radio arrays
- MARMOTS has recently been updated to include topography, which improves the predicted effective area by a factor of 1 to 3 (energy dependent) at the BEACON prototype site
- MARMOTS can be adapted for use by non-radio experiments
- To read more about MARMOTS, check out: **arXiv:2504.13271**